VOLUME LX NUMBER 2 WHOLE 526

SCHOOL SCIENCE AND MATHEMATICS

FEBRUARY 1960

School Science and Mathematics

A Journal for All Science and Mathematics Teachers

All matter for publication, including books for review, should be addressed to the editor. Payments and all matter relating to subscriptions, change of address, etc. should be sent to the business manager.

Entered as second class matter December 8, 1932, at Menasha, Wisconsin, under the Act of March 3, 1879. Additional entry at Oak Park, Illinois, January 18, 1957. Published Monthly except July, August and September at 450 Ahnaip St., Menasha, Wis. PRICE: Four dollars and fifty cents a year; foreign countries $5.00; current single copies 75 cents.

Contents of previous issues may be found in the Educational Index to Periodicals.

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ALL I CAN SAY HERE IS
3 BRICKS AND 4 NAILS.

$3b + 4n$

Unlike terms cannot be combined.

3 NAILS AND 4 NAILS
MAKE A TOTAL OF 7 NAILS.

$3n + 4n = 7n$

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SCHOOL SCIENCE AND MATHEMATICS

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Brief History of Computers*

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THE EARLIEST COMPUTERS

The first automatic digital computer was completed just fifteen years ago, so that one might suppose that the principles underlying its construction are of fairly recent origin. It is true that the engineering techniques had to await the coming of the electronic age, but all the ideas were fully developed more than a century earlier by Charles Babbage.

It was in 1812 that Babbage, then a student at Cambridge, proposed a difference engine which could calculate new mathematical tables and check old ones. At that time most tables were full of errors caused by human failings in computing and copying. For instance, the *Nautical Almanac*, first published in 1767, was an unreliable guide, making navigation hazardous beyond the vagaries of wind and tide.

The Difference Engine

The difference engine envisaged by Babbage would not only calculate the figures, but also would stamp them directly onto a copper engraver's plate by means of steel punches. Because no human would intervene, there would be no mistakes.

This machine was to use the well-known method of differences for tabulating polynomials. Suppose that we wish to construct a table

* Work done under the auspices of the U. S. Atomic Energy Commission.

giving the values for the equation $y=n^3-n^2$, where n is any positive integer. Our table will look like this:

n	n^3	n^2	n^3-n^2
0	0	0	0
1	1	1	0
2	8	4	4
3	27	9	18
4	64	16	48
5	125	25	100
6	216	36	180

It is obvious that the arithmetic soon becomes quite involved.

By using the method of differences, we can greatly simplify our calculations and have a ready check on their accuracy. This method can be understood by examining the following extension to our table.

n	n^3	n^2	n^3-n^2	D_1	D_2	D_3	D_4
0	0	0	0	0	4	6	0
1	1	1	0	4	10	6	0
2	8	4	4	14	16	6	0
3	27	9	18	30	22	6	0
4	64	16	48	52	28	6	0
5	125	25	100	80	34	6	0
6	216	36	180	114	40	6	
			294	154	46		
			448				
				200			
			648				

D_1 is the difference between a given entry in the (n^3-n^2) column and the entry directly below it. D_2 is the difference between a given entry in the D_1 column and the entry directly below it, and similarly for D_3 and D_4.

Now instead of working from left to right, we can continue to find values for n^3–n^2 by working from right to left and using addition only, as has been done below the broken line. Because most mathematical functions can be expressed as a power series, this method is very useful.

Moreover, we can quickly spot any mistake. Suppose that for 4^3-4^2 we had printed 46 instead of 48, the correct value. Then D_3 instead of being all 6's would read 6, 4, 8, 2, 6, 6. The difference engine could identify such errors in tables already published, by purely mechanical means.

In 1822 Babbage built a small working model of a difference engine which could tabulate a second-order polynomial to eight decimals. The following year he gained the support of the British government for a much larger machine, one with provision for 20 decimals and capable of tabulating a seventh-order polynomial! Such an instrument, requiring thousands of precision parts, might be likened to a colossal watch. Every cogwheel, gear, and lever had to be machined to extremely close tolerances, a technique not very far advanced then. At each step Babbage had to develop new methods and invent new tools, as well as solve a number of fundamental problems, like how to perform the "carry" in addition.

In the calculating machines which had been built up to that time, each carry was performed in sequence. Suppose that 1 is to be added to 99999. In the existing machines, the units wheel would turn only the tens wheel; the tens wheel would then turn the hundreds wheel, and so on. Recognizing that this method would be inordinately slow when dealing with large numbers, Babbage suggested an entirely new approach. In his scheme, known as the "anticipatory carry," all the wheels locked and turned at once, thus effecting a great time saving. This method has been retained in many modern calculating machines.

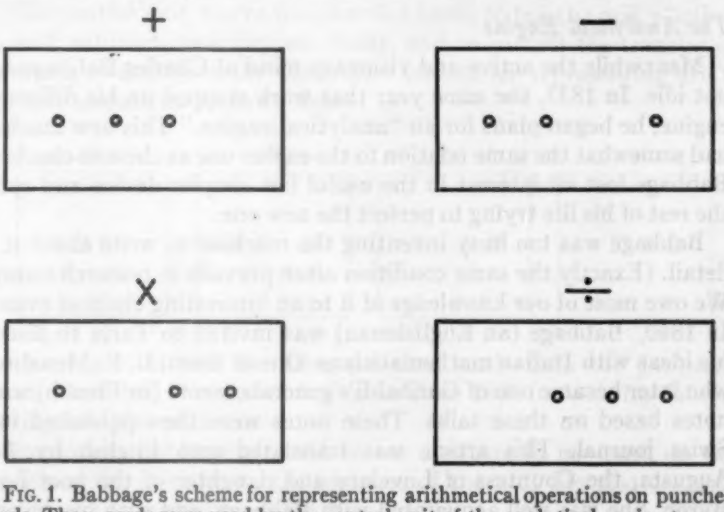


FIG. 1. Babbage's scheme for representing arithmetical operations on punched cards. These cards were used by the control unit of the analytical engine to control the transfer of numbers to and from the store.

The most revolutionary feature of the difference engine, though, was its ability to perform a controlled sequence of additions, and not just one at a time. When we consider the state of the art then existing, we realize the magnitude of Babbage's genius and ambition. The first mechanical adding machine had been built as long before as 1642, by Blaise Pascal, but it was not until near the end of the nineteenth century that a really satisfactory desk calculator was perfected. It is small wonder, then, that Babbage encountered enormous difficulties.

Notable progress was made on the difference engine, but still it was not completed by 1833. In that year a quarrel over money caused Babbage's chief engineer to resign, taking with him all the specialized tools they had developed. As a result, the government withdrew its aid and work stopped. In all, the government had spent 17,000 pounds, a huge sum in those days.

Twenty years later, in 1853, Georg and Edvard Scheutz of Stockholm demonstrated the soundness of Babbage's concepts. They succeeded in building a smaller version of a difference engine based on his design. It could tabulate to 14 decimals a fourth-order polynomial. After being exhibited in London and Paris in 1855, this machine was purchased for $5000 by an American businessman and given to the Dudley Observatory in Albany, N. Y. A replica of this difference engine was built in 1858 for the British government and used in 1864 to prepare a set of life tables. This replica now resides in the Science Museum at South Kensington, London.

The Analytical Engine

Meanwhile the active and visionary mind of Charles Babbage was not idle. In 1833, the same year that work stopped on his difference engine, he began plans for an "analytical engine." This new machine had somewhat the same relation to the earlier one as chess to checkers. Babbage lost all interest in the useful but simpler device and spent the rest of his life trying to perfect the new one.

Babbage was too busy inventing the machine to write about it in detail. (Exactly the same condition often prevails in research today.) We owe most of our knowledge of it to an interesting chain of events. In 1840, Babbage (an Englishman) was invited to Turin to discuss his ideas with Italian mathematicians. One of them, L. F. Menabrea, who later became one of Garibaldi's generals, wrote (in French) some notes based on these talks. These notes were then published in a Swiss journal. This article was translated into English by Ada Augusta, the Countess of Lovelace and daughter of the poet Lord Byron. She was well acquainted with Babbage, and with his encouragement wrote extensive notes on the notes, which are today our

primary source of information on the analytical engine. Later, she and Babbage collaborated in devising an "infallible" system of betting on horse races, the end result of which was that Lady Lovelace had to pawn the family jewels and leave them "in hock" for several years.

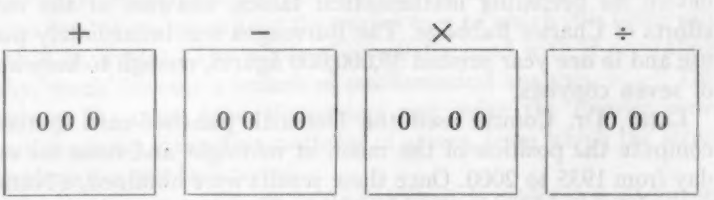
In principle the analytical engine had all the features we recognize in a modern automatic digital computer. The main difference lay in the fact that today we use electronic circuits, whereas Babbage had only mechanical mechanisms at his disposal.

The analytical engine could perform any mathematical operation and do it automatically in the proper sequence, or as we now say, "carry out a program." At intermediate steps it could choose the best of several alternatives presented it. It would then change the instructions for its subsequent operations. It could be made to go back and cycle over any part of a computation. It was to have a *store* for numbers, a *mill* for performing arithmetic operations on the numbers, a *control unit*, and *input* and *output* devices, or in other words, all the elements of a computer.

For the store (memory) Babbage intended to use 1,000 columns of 50 counting wheels each. Thus, the machine would have a capacity of 1,000 fifty-digit numbers! It was typical of his quixotic nature that he should try to attain the stars without first achieving the moon. For comparison, Mark I, the first modern computer, contains only 60 constant registers and 72 adding registers.

The mill (arithmetic unit) would employ the principle of the anticipatory carry developed earlier for the difference engine.

The control unit was to use punched cards to tell the mill whether to add, subtract, multiply, or divide, and to control the transfer of numbers to and from the store. The scheme for representing arithmetical operations is shown below.



Babbage got his idea for using punched cards from the Jacquard loom, which had been in use in France since 1804 for weaving fabrics having complex patterns.

For input, Babbage planned to set numbers in by hand on the wheels of the store or on the registers of the mill. He also contemplated using punched cards for this.

As output, the machine would (a) print directly on paper, (b)

stamp a stereotype mold, or (c) punch cards. No one would have to laboriously copy its prodigious output.

Babbage estimated that the analytical engine could perform sixty additions or subtractions a minute. It could multiply 50 decimals by 50 decimals, or divide 100 decimals by 50 decimals in one minute. The time required for multiplication or division could be reduced providing the numbers contained fewer digits. While this is hopelessly slow compared to modern high-speed computers, in which these operations are done in a matter of microseconds, it was a phenomenal speed then.

Unfortunately, the analytical engine was never finished, although Babbage spent large sums of his own money in financing it and left thousands of detailed drawings of its parts. He died an unappreciated and embittered man. The very existence of his analytical engine was as forgotten by the world as the Boro Budur or King Tut's tomb. Only in the past decade was his genius rediscovered.

Later Developments

The use of punched cards, an offshoot from the Jacquard loom, was not forgotten. In 1886 the data from the United States census of 1880 were still being sorted and counted. Dr. H. Hollerith, director of the census bureau, saw that unless a new approach were used, the task would not be finished before the 1890 census began. He therefore devised a system of recording the census information on punched cards and invented machines to sort them and tabulate the data.

In the late 1920's Dr. L. J. Comrie of the *Nautical Almanac* Office in England discovered that a Burroughs calculating machine (an American business machine) could be used without modification as a difference engine. Thus Dr. Comrie stumbled onto a ready-made device for preparing mathematical tables, unaware of the earlier efforts of Charles Babbage. The Burroughs was immediately put to use and in one year printed 30,000,000 figures, enough to keep ahead of seven copyists.

Later, Dr. Comrie used the Hollerith punched-card system to compute the position of the moon at midnight and noon for every day from 1935 to 2000. Once these results were obtained, a National accounting machine was used to fill in the values for each hour.

Mark I—the First Modern Computer

Many improvements in calculating machines and punched-card techniques were made in the years following Babbage's death in 1871, but apparently no one thought of designing an automatic digital computer until 1937, when such an idea occurred to Howard Aiken of Harvard. Aiken enlisted the support of the International Business

Machines Corporation, and during the next seven years they built Mark I, or "Automatic Sequence Controlled Calculator." It was presented to Harvard University on August 7, 1944, a gift from IBM.

Essentially, Mark I is a mechanical device, although it uses electromagnetic relays and electric motors. It can perform three additions per second and works to 23 significant figures. Though sluggish by present standards, Mark I proved very useful in performing ballistic calculations and preparing mathematical tables of all kinds. What is more important, it initiated the development of modern computers.

Introduction of Electronic Circuits

The first nonmechanical computer was the ENIAC (Electronic Numerical Integrator and Computer). Completed in 1946 at the Moore School of Electrical Engineering of the University of Pennsylvania, ENIAC contains 18,000 vacuum tubes and 1,500 relays. It can do addition more than 1000 times as fast as Mark I, but has an even smaller storage.

There followed in rapid succession a whole array of computers with names like EDSAC, EDVAC, and UNIVAC. While increasing in speed and storage capacity they have grown physically smaller, owing to refinements in design and the replacement of vacuum tubes by tiny transistors. Present research is directed toward even further improvements, particularly in regard to high-speed memories and output and input devices.

In connection with the latter, an instrument resembling a television camera is being perfected which can read a printed page and transcribe the language into electrical pulses. If translation of foreign languages by computers is ever to be practical, such a device is indispensable.

Babbage would certainly be surprised at the form his analytical engine has taken, but not by the varied uses to which it is being put. In 1838 he wrote: "The whole of chemistry, and with it crystallography, would become a branch of mathematical analysis. . . ." This prophecy has come true. Computers now solve the Fourier series describing x-ray diffraction patterns of atoms, from which the structure of matter can be inferred.

Other routine uses include analyzing particle orbits in a cyclotron, calculating safe wing stresses for aircraft, and predicting the weather. Recently someone even suggested building a "Presidential computer," one which could make important decisions of state based on logical premises. It would determine such things as the probability of a Russian attack at any moment and recommend the proper course of action. Insofar as possible, the President's own values would be put into the computer as a set of propositions, even though these would

admittedly be inconsistent. The computer would somehow correlate these and presumably come up with the "right" decision every time. (If you are skeptical about the merit of this proposal, you are not alone, since that is the authors' attitude.)

Man has assembled a machine which increases the speed and accuracy of computations while relieving him of the drudgery, or which can play a safe but not brilliant game of chess. It has no originality and no sense of values. The question of whether mankind as a whole has gained or lost in these qualities is an interesting one which cannot be decided by a machine.

Queries Concerning the Eyes II

John Satterly
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1. As a boy I would often lie on the turf in a cricket field on a warm summer day and gaze up at the sky. My field of view seemed to be occupied by dancing particles. They were not dust motes. I was beginning the study of physics and chemistry and in my imagination these particles were the molecules of air. In later years I thought the effect might be a kind of Brownian motion either in the air or possibly in the liquids of my own eyes. What is the correct explanation?

2. If one looks directly at the sun with half closed eyes or at the reflection of the bright sun in a convex mirror the appearance is that of the bright-nearly point-source with a wide fringe of (coplanar) rays radiating in all directions. Any one ray is seen to be brightly colored in patches which have blue and yellow end portions; suggestive of the spectra of different orders as seen in a coarse grating. Is this partly a diffraction effect produced by the eye lashes? The eyelashes are however practically parallel and these radiating rays are in all directions.

3. If one closes the eye and presses, with the finger, the corner of the eye ball next to the nose there appears at the opposite end of the eye a brightly colored patch like a bright-broad ring. I supposed the effect is due to nerve stimulation by pressure. I was surprised to learn that some of my younger colleagues who are well up in quantum mechanics, mesons and such-like had never observed this effect. Its production is different from the negative and positive after-effects which appear after looking at a bright object and subsequently closing the eyes. These have been described often.

4. I find that I can produce very vivid color effects within the eyes by mere slight frontal pressure upon the eye-balls such as may happen during prayers in a half-darkened church. A bright broad elliptical ring appears in the eye. If the ring is blue the interior is yellow. Quickly the colors may interchange and do so more than once; ultimately they fade away. Initially the colors appear to be more glorious than the colors of external objects. This same effect is often noticed when on rising from my bed and going into my darkened study (blinds perhaps not fully down), and this without any pressure being applied to the eyes; perhaps change of bodily position has done the trick.

I may add that I am partly color blind; red-blind is a short description. I always make mistakes when asked by normal-eyed people to name the colors of red, green and brown objects. To me the spectrum of a white light is made up of just two colors blue and yellow, the so-called green in the middle being to me nearly white and the red just a dark yellow.

Ceramic Engineering Education for Tomorrow's High School Graduate*

Ralph L. Cook
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The field of ceramic engineering offers excellent opportunities for the high school graduate. In addition to the usual fields such as glass, porcelain enamels, refractories and structural clay products, the ceramic engineering graduate finds challenging opportunities in the fields of electronic components, materials for missiles and rockets, and in nuclear and atomic applications. Ceramic engineering might be thought of as a field of high temperature engineering.

High School Preparation

The best high school preparation for entrance into the field of ceramic engineering is a thorough background of training in mathematics, physics and chemistry. The general requirements for entrance into the curriculum of ceramic engineering are the same as for any field of engineering. It is to be emphasized that the curriculum in ceramic engineering is a strong engineering curriculum and is administered by the College of Engineering. The engineering educational requirements at the University of Illinois are 3 units of English, 2 units of Algebra, 1 unit of Geometry and ½ unit of Trigonometry. High School students who have only 1 unit of Algebra and 1 unit of Geometry may be admitted on condition that the deficiency in Algebra and Trigonometry be removed during the first year. In addition to these entrance requirements, the College of Engineering recommends that the following courses be included: one additional unit in English, 2 units in a language, 2 units in Science including Physics or Chemistry or both, ½ unit in Solid Geometry and 2 units in Social Studies.

Ceramic Engineering Curriculum

The specific curriculum in Ceramic Engineering for the Freshman year at the University of Illinois is identical with the other fields of engineering. All Freshman engineers take the same curriculum the first year. These courses include two semesters of Rhetoric and English Composition, Analytical Geometry and Calculus, Engineering Drawing and Engineering Geometry, normally two semesters of Inorganic Chemistry and the first course in college physics the second semester. In addition, courses in military and physical educa-

* Presented at the Chicago Section Meeting of the American Ceramic Society, October 23, 1959. The High School Counselors from the Chicago Area were invited to this meeting on Ceramic Education.

tion are required both in the Freshman year and in the Sophomore year.

With some students it may not have been possible to obtain the necessary college algebra and trigonometry in high school. If this is the case, these courses can be taken the first semester, deferring the required work in mathematics until later in the curriculum. With respect to Chemistry, some choices are possible. If one had Chemistry in high school and graduated in the upper 25 per cent of the class, it is permissible to take an advanced chemistry course, Chemistry 109, for five hours credit, completing in one semester the major portion of the work covered in the two semesters of chemistry. For those who have not had chemistry in high school, a special course is available in the first year.

In the Sophomore year of the curriculum in Ceramic Engineering the emphasis continues to be placed on the fundamental courses in engineering. The work in physics is continued both semesters and includes work in modern and nuclear physics. The course in integral calculus is required and additional work is encouraged in advanced calculus and differential equations. A course in Quantitative Analysis is required the first semester and a course in Analytical Mechanics which includes both statics and dynamics the second semester. In addition, a course in Economics and a course in Mineralogy are required, as well as a specific course each semester in Ceramic Engineering. These two courses cover the properties and structure of ceramic materials and ceramic processes and equipment.

In the Junior year the work in fundamental engineering is continued with courses in Resistance of Materials both lecture and laboratory and Structural Engineering. The work in chemistry is continued with a required course in physical chemistry. Along with these basic or fundamental courses in engineering and chemistry, emphasis is placed on the fundamentals in ceramic engineering which includes such courses as ceramic technology, the study of structure, reaction mechanisms, and phase relations of ceramic material the first semester, and the properties of particles and particle aggregates the second semester. The other courses on ceramic fundamentals cover the principles and mechanism of drying as well as the firing process, utilization of fuels, kilns and furnaces and their operation. These four courses are similar to the unit operation courses of the chemical engineering curriculum. In addition to the fundamental courses in ceramic engineering in the Junior year, specific courses are required in Pyrometry—instrumentation and measurement, in Porcelain Enamels and in Glass. Also in the third year at least a three hour course in Social Sciences—Humanities is required each semester.

In the Senior year a three hour course in Basic Electrical En-

gineering is required the first semester followed by a second course in either Electronics or Motor Control and Power Equipment. One course each semester is required in Ceramic Engineering Design. In the first semester a course is required in Whiteware Bodies and Glazes and in the second semester the Refractories Technology course. These four courses account for about one half of the work in the Senior year; the balance is selected from elective courses. The elective courses are divided into two general types—technical and non-technical. The technical elective courses may be in the field of ceramic engineering and include additional emphasis in glass, procelain enamels, electrical ceramics or ceramic microscopy. Many of the ceramic engineers select technical electives in the field of advanced mathematics or chemistry, in solid state physics or other fields of engineering. About 20 per cent of the four year curriculum in ceramic engineering is devoted to non-technical electives which include the courses in social science and humanities.

The curriculum in ceramic engineering is designed, first, to give a good foundation in engineering in general and the fundamental or basic aspects of ceramic engineering, second, to give an opportunity for specialization in some phase of ceramic engineering and third, to give an opportunity for selection of courses in engineering science to serve as a foundation for later graduate work.

Opportunities for Graduate Ceramic Engineers

There are extensive opportunities for the ceramic engineering graduate in a five billion dollar growing industry. These opportunities may be in (1) Production (2) Management and Sales (3) Engineering Control or (4) Research and Development. There has been a wide choice of positions in industry for ceramic engineering graduates. Many times the graduate has had his choice from as many as fifty different positions. Over the past ten years the starting salary of the ceramic engineer with a B.S. degree has increased steadily until in June, 1959 it was in excess of $525.00 per month.

Approximately 25 per cent of the graduates elect to take additional graduate work leading to the Master's or Doctorate degree. Several times during the past ten years there have been more Fellowships available than qualified applicants. These graduate Fellowships carry a stipend of $1500–$2000 for the nine month academic year.

Scholarships

There are a large number of general scholarships for qualified high school graduates enrolling in the University. These scholarships include the General Assembly Scholarships, County Scholarships, and the Illinois State Scholarship. In addition, there are a limited num-

ber of special Ceramic Scholarships amounting to $200.00 for the first year and $300.00 for the second year. Also, this past year the Aero-jet Corporation of California gave a four year scholarship amounting to $500.00 each year to the outstanding entering Freshman in Ceramic Engineering. The Owens-Corning Fiberglas Scholarship carries an award of $500.00 to an outstanding Junior in Ceramic Engineering. The Alcoa Foundation makes an award of $500.00 to a Senior in Ceramic Engineering. The Pennsylvania Glass Sand Corporation awards a tuition scholarship for the senior year for the student in Ceramic Engineering who has the highest average for the Junior year's work. In addition, there are several other scholarships of a more general nature available to the upperclassman. In addition to the scholarships, there is frequently an opportunity for part time work on one of the various sponsored research programs.

The present scholarship program consisting of $200.00 for the first year and $300.00 for the second year seemed to be very effective in interesting well qualified high school seniors. Approximately fifteen such scholarships were awarded this past year. These scholarships are financed by contributions from industry and alumni. In order for such a scholarship program to be continued, the active support of the ceramic industry and individuals in the ceramic field will be required.

Enrollment

The enrollment in ceramic engineering reached a high level in the years immediately after the end of the war at the University of Illinois. There was a general decline in enrollment in the period 1950 to 1955 in ceramic engineering as in other fields of engineering. A year ago this fall the number of Freshmen in Ceramic Engineering decreased to nine with a total of 42 undergraduates. This year there are 23 in the Freshman class and a total of 72 undergraduates. During the past ten years the graduate enrollment has remained at a good level between 15 and 25.

The appreciable increase in the undergraduate enrollment this year is due both to the increase in the number of entering Freshmen and to a substantial increase in tne number of students transferring to the ceramic engineering curriculum from other departments. The increase in the Freshmen enrollment is due both to the cooperation of the high school counselor and to the availability of scholarships.

The total enrollment in engineering at the University of Illinois has remained near 3800 for the last four years. Actually there is a decrease of about six per cent in the total enrollment this fall as compared with a year ago.

Dramatizing Mathematics

Margaret F. Willerding
San Diego State College, San Diego, California

There exists a need for a broad, usable bibliography of dramatization on mathematical subjects. Many teachers in the Junior and Senior High Schools would like to "dramatize mathematics" either in the classroom as a teaching project or for a P.T.A., Mathematics Club or Assembly Program, but lack the time either to write the script themselves or have the class write it as a class project. This bibliography has been compiled to fill this need.

Children at the Junior High School level are very active. They are spontaneous, they lack self-consciousness, and they enjoy "make believe." Mathematics is dynamic, and children should appreciate its activity and be stimulated by it. Dramatization can be the ground where the active child and the active subject meet.

The average High School Mathematics Club cannot run itself. Club meetings require the preparation of interesting programs along mathematical lines. Thus there persistently arises the question, "What shall we do?"

It is hoped that this bibliography will help fill the needs of both of these groups.

Unless otherwise stated, the plays, pageant, assembly programs, etc. in the bibliography are for senior high schools. In the bibliography, volume and page numbers are given. Abbreviations used are:

AMM *American Mathematical Monthly*
MT *The Mathematics Teacher*
SSM SCHOOL SCIENCE AND MATHEMATICS

BIBLIOGRAPHY

PLAYS, PAGEANTS AND SKITS

"A Mathematical Playlet," *MT*, Vol. XXXVIII, 1945, pp. 309–313.
A costume play depicting the use of mathematics in business, science, art, music, etc.

ANNING, N., "Socrates Teaches Mathematics," *SSM*, Vol. XXIII, 1923, pp. 581–584.
Short play in one act. Explains the expansion of $(x+y)$ and $(x-y)$ through the use of blocks.

BROWN, I. M., "Adventures of an *X*," *Open Court*, Vol. XXVIII, 1917, pp. 529–537.

BROWNELL, E., "Mathesis," *MT*, Vol. XX, 1927, pp. 459–465.
A simple play with fifteen characters. Suitable for Junior High School.

CLARK, M., *et al.*, "Adventures of *X*," Pamphlet, D. C. Heath Co., Boston, 1916.

COHEN, DENA, "A Problem Play," *MT*, Vol. XXIX, 1936, pp. 78–83.
A play in which King Math of Numberland helps Alice solve a fraction problem. Suitable for Junior High School.

CORDELL, CHRISTOBEL, "Domestic Dilemma," *Colorful Teaching of Mathematics*, J. Weston Walch Publ. Co., Portland, Maine, 1955, pp. 41.
A play depicting the usefulness of mathematics for girls. Suitable for Junior High School.

CORDELL, CHRISTOBEL, "Dramatized Problems," *Colorful Teaching of Mathematics*, J. Weston Walch Publ. Co., Portland, Maine, 1955, pp. 19.
Dramatized problems. May be used for Mathematics Club program or as class presentation. Suitable for Junior or Senior High School.

CORDELL, CHRISTOBEL, "No More Math!" *Colorful Teaching of Mathematics*, J. Weston Walch Publ. Co., Portland, Maine, 1955, pp. 25.
Depicts necessity for knowledge of general mathematics in almost any type of job. Suitable for Junior and Senior High School.

CORDELL, CHRISTOBEL, "Pageant of Numbers," *Colorful Teaching of Mathematics*, J. Weston Walch Publ. Co., Portland, Maine, 1955, pp. 3.
Dramatization in six scenes depicting history of mathematics. May be used in its entirety or separately by scenes. Scenes 1, 2, and 3 suitable for Junior High School.

CORDELL, CHRISTOBEL, "Problem Child," *Colorful Teaching of Mathematics*, J. Weston Walch Publ. Co., Portland, Maine, 1955, pp. 31.
Play demonstrating the difficulties that may eventually be encountered by the student who fails to study mathematics. Junior or Senior High School.

COWLEY, E., "Mathematical Fantasy," *SSM*, Vol. XXXIV, 1934, pp. 240–241.
Short skit showing that numbers are indispensable in the modern world. Fourteen players.

CRAWFORD, ALMA E., "A Little Journey to the Land of Mathematics," *MT*, Vol. XVII, 1924, pp. 336–340.
A play in which two high school pupils get a vision of the value of mathematics in chemistry, drafting, bookkeeping, etc. Twenty-six characters. Suitable for Junior High School or ninth grade.

DENBOW, C., "Traders and Trappers," *MT*, Vol. XXXIV, 1941, pp. 61–65.
Deals with the principle of place value and number concepts in arithmetic. Junior High School.

FLETCHER, R., "Quarter After Ten—A Mathematics Play in One Act," *Texas Outlook*, Vol. XV, 1931, pp. 31–35.
A play in one act.

FORREST, L., "Euclid, Agrarian Arbitrator," *MT*, Vol. XXV, 1932, pp. 22–26.
Based on the concept of congruent triangles. Suitable for geometry classes. Four characters.

FORT, TOMLINSON, "Mechanics—A Dramatic Skit," *AMM*, Vol. XXXVIII, 1931, pp. 42–46.
Depicts the relation between mathematics and physics. Nine characters. Suitable for college.

GOWAN, J. C., "Mathematics Skit," *MT*, Vol. XXXVI, 1943, pp. 169–170.
Short play telling how mathematics is used in everyday life.

GRAFF, MARGARET, "Euclid Dramatized," *SSM*, Vol. XXI, 1921, pp. 381–386.
Dramatization of stories of early mathematicians.

GRAY, MRS. JAMES T., "Historically Speaking," *SSM*, Vol. LII, 1952, pp. 345–355.
History of measures and zero. Time: 30 minutes. Fourteen characters. Suitable for Junior High School.

HALL, A., "A New Assistant," Pamphlet, Metric Assoc., Pottsville, Pa., 1938.
Deals with the spread of the metric system throughout the world. Fifteen characters, ten speaking parts.

HARDING, P. H. *et al.*, "A Mathematical Victory," *SSM*, Vol. XVII, 1917, pp. 475–482.
A rather disjointed play for mature students.

HARMAN, MILDRED MARIE, "All in One Day," *SSM*, Vol. LV, 1955, pp. 15–23.
Play dealing with mathematics in everyday living. Eight characters. No properties.

HATCHER, FRANCES, B., "A Living Theorem," *SSM*, Vol. XVI, 1916, pp. 39–40.
A short skit.
HATTON, M., and SMITH, D., "Falling in Love with Plane Geometry," *MT*, Vol. XX, 1927, pp. 389–409.
A clever comedy. Sixteen characters.
JACKSON, *et. al.*, "Story of Mathville," *Nations' Schools*, Vol. XXIII, 1939, pp. 27–28.
KARAPETOFF, VLADIMIR, "Agha and Math," *Scripta Mathematics*, Vol. XII, 1946, pp. 153–159.
Fantasy on the way logs might have been invented even though they weren't. Suitable for High School and College.
KIDD, KENNETH, "How Much?—A Mathematics Playlet," *SSM*, Vol. LIV, 1954, pp. 628–634.
Depicts how man from earliest times passed through certain steps in development of ways to answer the question "how much?"
LEPS, J., "The Radical Dream: A Mathematical Play for Puppets," *SSM*, Vol. XXXII, 1936, pp. 644–647.
A play about algebra.
LESKOW, OLIVE, "The Cubic," *MT*, Vol. XXXVI, 1943, pp. 312–316.
Depicts contribution to mathematics of Cardan, Tartaglia and Ferrari. Eight characters.
LLOYD, DANIEL, "In Quest of Truth," *SSM*, Vol. LI, 1951, pp. 275–285.
Historical pageant in two parts. Part I suitable for Junior High School. Part II suitable for Senior High School and College.
McSORLEY, K., "Mock Trial of B Versus A," *SSM*, Vol. XVIII, 1918, pp. 611–621.
Clever dramatization based on S. Leacock's "*A, B, C.*"
Mathematics Department of Deering High School, "Math Around Us," *MT*, Vol. XLVI, 1953, pp. 260–261.
Short play in verse about geometry and trigonometry. Four speaking parts.
MEALS, LOUISE, "*X* on the Spot," *MT*, Vol. XXXII, 1939, pp. 296–300.
A play depicting the practical value of learning mathematics. Six characters. Suitable for Junior and Senior High School.
MILLAY, K., AND FREE, L., "Flatlanders—Mathematics Play in One Act." *Scholastic*, Vol. XXVI, 1935, pp. 9; *AMM*, Vol. XXVI, 1919, pp. 264–267; *SSM*, Vol. XIV, 1914, pp. 583–587, *Mathematics Gazette*, Vol. VII, 1914, pp. 228–231.
Play for four characters based on Abbott's *Flatland*.
MILLER, FLORENCE BROOKS, "An Idea that Paid," *MT*, Vol. XXV, 1932, pp. 472–479.
Play in three acts about profit, loss, partnerships, corporations and stocks. Suitable for Junior High School.
MILLER, FLORENCE BROOKS, "A Near Tragedy," *MT*, Vol. XXII, 1929, pp. 472–481.
A play in three scenes showing the effect of having no mathematics. Suitable for Junior High School.
MILLER, FLORENCE BROOKS, "Out of the Past," *MT*, Vol. XXX, 1937, pp. 366–370.
Development of the number system and the use of mechanical devices for computation. Suitable for ninth grade.
MOSSMAN, E., "Mathematics Pageant," Pamphlet, Berkeley, Calif., Garfield Junior High School, no date.
NOVINGER, F. F. *et al.*, "I Had the Craziest Dream," *MT*, Vol. XL, 1947, pp. 20–23.
Depicting the practical applications of mathematics. Fifteen characters. Suitable for Junior High School.
PARKYN, HANNAH, "When Wishes Come True—A Mathematical Play," *MT*, Vol. XXXII, 1939, pp. 16–24.
A play for thirty-three players about algebra and geometry.

PATERSON, E., "Every Man's Visit to the Land of Mathematicians," *MT*, Vol. XXXI, 1938, pp. 7–18.
A fairly elaborate play about mathematics from ancient to modern times.
PITCHER, W. E., "Alice in Dozenland," *MT*, Vol. XXVII, 1934, pp. 390–396.
Use of Duodecimal system. Five characters.
PITCHER, W. E., "The Mathematics Club Meets," *MT*, Vol. XXIV, 1931, pp. 197–207.
A play for sixteen players depicting methods of computation among Egyptians, Romans, Italians, and Russian Peasants.
POTWINE, E. B., "Bobby Learns His Numerals," *MT*, Vol. XXXVI, 1943, pp. 29–33.
History of numbers. Eleven characters. Calls for the preparation of fifteen charts. Suitable for Jr. and Sr. High School.
RAFTERY, GERALD, "The Eternal Triangle," *MT*, Vol. XXVI, 1933, pp. 85–92.
A play about indirect measure for Junior High School and ninth grade. Three characters.
REID, N., "Archimedes, A Mathematical Genius," *SSM*, Vol. XLI, 1941, pp. 211–219.
Contributions of Archimedes. Outline only, lines supplied by players.
RUSSELL, C., "More Than One Mystery," *MT*, Vol. XXVI, 1933, pp. 477–481.
Play based on puzzle problems. Seven players. Suitable for Junior High School.
SCHLAUCH, T., "Point College," *SSM*, Vol. XXXI, 1931, pp. 448–454.
Play concerning Analytic Geometry. Players are animated points. Fifteen characters. Suitable for college.
SCHLIERHOLZ, T., "A Number Play in Three Acts," *MT*, Vol. XVII, 1924, pp. 154–169.
Dramatization of D. E. Smith's *Numerous Stories of Long Ago*. Suitable for Junior High School.
SCOTT, E., "Geometry Humanized," *MT*, Vol. XXI, 1928, pp. 92–101.
Clever play about geometry. Eighteen characters. Simple properties.
SHEA, JOHN, "Mr. Chips Teaches Geometry," *SSM*, Vol. XL, 1940, pp. 720–726.
A play for nine players. About Locus concept.
SKERRETT, JOSEPHINE, "A Mathematical Nightmare," *MT*, Vol. XXII, 1929, pp. 413–317.
A play depicting what happens if there are no mathematics. Ten players. Suitable for Junior High School.
SLAUGHT, H., "Evolution of Numbers—Historical Drama in Two Acts," *MT*, Vol. XXI, 1928, pp. 305–315; *AMM*, Vol. XXXV, 1928, pp. 146–151.
History of the number systems of algebra.
SMITH, ALICE, "Case of Matthew Mattix," *MT*, Vol. XXVI, 1933, pp. 286–291.
A comedy in two scenes.
SMITH, ALICE, "Snow White and the Seven Dwarves," *MT*, Vol. XXXVII, 1944, pp. 26–30.
A play in three acts about arithmetic and geometry. Eleven players. Suitable for Junior and Senior High School.
SNYDER, RUTH L., "If," *MT*, Vol. XXII, 1929, pp. 482–486.
Tells the importance of mathematics. Two acts, nine players.
SPERKS, MARIE, "The Determination of Taxes in the Community," *SSM*, Vol. XLII, 1942, pp. 454–462.
A play for Junior High School that can be used with a unit on taxation.
SPITZER, HERBERT, "How We Got the Name 'Counter,'" *Teaching of Arithmetic*, Houghton-Mifflin Co., N. Y., 1948, pp. 354–356.
Skit dramatizing how the word "counter" came into use. Three characters. Suitable for Junior High School.
STARK, M., "Modern Mathematics Looks Up Its Ancestors," *AMM*, Vol. XLIII, 1936, pp. 299–304.
A play about history of mathematics. Written in blank verse.
TAYLOR, HELEN, "Assembly Sketch," *SSM*, Vol. XXX, 1930, pp. 631–632.

Skit on improvement of mathematics in high school.

TURNER, ETHEL, "The Mathematics News," *MT*, Vol. XLVII, 1954, pp. 249–253.
Clever play depicting mathematics in the news. Time: 40 minutes. Eighteen or more characters. Suitable for Junior and Senior High School and college.

WARD, J. A., "A Wax Works Show," *MT*, Vol. XXXIV, 1941, pp. 266–268.
A skit on Newton, Einstein and Archimedes.

WHELAN, A., "It Can't Happen Here," *AMM*, Vol. XLV, 1938, pp. 617–628.
A musical farce on "why study math in college." Six principal characters and chorus.

WHITAKER, H., "Mathematical Quest," *SSM*, Vol. XX, 1929, pp. 457–459; *MT*, Vol. XVIII, 1925, pp. 356–358.
A pageant in verse. Fifty characters, two speaking parts.

WILLERDING, MARGARET F., AND NIES, RUTH H., "An Assembly Program on Weights and Measures," *School Activities*, Vol. XXVII, 1955, pp. 63–66.
Pantomine on the history of weights and measures. One speaking part. Fairly elaborate.

WILLERDING, MARGARET F., "At Arm's Length," Original Radio Script from the Program Series *A Full Measure of Adventure*, Station K.S.L.H., St. Louis, Missouri.
Story of the historical development of the "yard."

WILLERDING, MARGARET F., AND NIES, RUTH H., "Dorothy and the Wizard of Arithmetic," *School and Community*, Vol. XXXIX, 1953, pp. 11–12.
Take-off on the Wizard of Oz. Twenty-six characters.

WILLERDING, MARGARET F., "Dramatizing an Arithmetic Trick," *School and Community*, Vol. XXXVI, 1950, pp. 318–319.
Skit showing pupils a short way to find a percentage grade from a common fraction.

WILLERDING, MARGARET F., "The Egyptian Rope Stretchers," Original Radio Script from the Program Series *A Full Measure of Adventure*, Station K.S.L.H., St. Louis, Missouri.
Story of the Egyptian Rope Stretchers and their methods of marking right angles.

WILLERDING, MARGARET F., AND NIES, RUTH H., "How Much? How Many? How Heavy?" *Grade Teacher*, Vol. LXXIII, 1955, pp. 53.
A pantomine on the history of weights and measures. Fairly elaborate costuming.

WILLERDING, MARGARET F., "The Human Measuring Stick of Long Ago," Original Radio Script from the Program Series *A Full Measure of Adventure*, Station K.S.L.H., St. Louis, Missouri.
Story of the human measures such as the cubit, span, hand, palm, etc.

WILLERDING, MARGARET F., "Once in a Blue Moon," Original Radio Script from the Program Series *A Full Measure of Adventure*, Station K.S.L.H., St. Louis, Missouri.
Story about the superstitions connected with numbers.

WILLERDING, MARGARET F., "Rock About His Neck," Original Radio Script from the Program Series *A Full Measure of Adventure*, Station K.S.L.H., St. Louis, Missouri.
Story of the Apache method of counting.

WILLERDING, MARGARET F., "Sizing the Land," Original Radio Script from the Program Series *A Full Measure of Adventure*, Station K.S.L.H., St. Louis, Missouri.
History of the land measures, acre, furlong, and rod.

WOODARD, MARY, "A Study in Human Stupidity," *MT*, Vol. XXXVIII, 1945, pp. 362–367.
Story of life of Galois.

WOOLERY, FREDA, "Much Ado About Mathematics," *MT*, Vol. XXXVIII, 1945, pp. 23–35.
Portrays the life of a person who has been forbidden to use mathematics.

QUIZ SHOWS AND DRAMATIZED ASSEMBLY PROGRAMS

ADLER, IRVING, "Fun With Mathematics," *MT*, Vol. XLII, 1949, pp. 153–155.

ADLER, IRVING, "Theory and Practice," *MT*, Vol. XLI, 1948, pp. 218–220.

CHAPMAN, HOPE, "A Mathematical Quiz Program," *MT*, 1953, pp. 537–540.

DIGGINS, JULIA, "A Mathematical Dr. I-Quiz-Em Program," *MT*, Vol. XLV, 1952, pp. 30–33.

SCHOR, HARRY, "A Mathematics Assembly Program," *MT*, Vol. XLVII, 1954, pp. 476–478.

VAUGHAN, A., "Professor Whiz and His Class in Math Magic," *SSM*, Vol. XXXIX, 1939, pp. 540–545.

WILLERDING, MARGARET F., AND NIES, RUTH H., "An Assembly Program on Weights and Measures," *School Activities*, Vol. XXVII, 1955, pp. 63–66.

ANNUAL NSTA CONVENTION

The eighth annual convention of the National Science Teachers Association will be held in Kansas City, Missouri, March 29–April 2, 1960.

All sessions will emphasize evaluation and improvement in "Current Science and the K-12 Program," the convention theme. General sessions and the Exposition of Science Teaching Materials by producers and publishers will be held in the Kansas City Municipal Convention Center. Banquet and luncheon sessions and other related meetings are scheduled in the two convention headquarters hotels, the Muehlebach and the Phillips.

Current science topics will be reported by college and research scientists speaking on recent developments in their respective fields. Two Nobel prize winners head the roster of speakers: Dr. Linus C. Pauling, Nobel Laureate, Professor of Biochemistry, California Institute of Technology (Pasadena) and Dr. Walter H. Brattain, Nobel Laureate, Physical Research Department, Bell Telephone Laboratory (Murray Hill, New Jersey). Other research speakers include Dr. George B. Kistiakowsky, Science Advisor to President Eisenhower and Dr. John R. Heller, Director, U. S. National Cancer Institute.

SPACE SATELLITES

The orbit of a space satellite is the most sensitive known tool for studies of the earth and its atmosphere. Like the super-sensitive needle of a finely calibrated instrument, the path of a satellite responds to every change in the "pull" of gravity. And so artificial moons become man's best method for determining the earth's shape, size, mass, and the distribution of materials that make up its crust.

Gravity is far from uniform. Mountain ranges, ocean basins, unevenly distributed masses in the earth's crust, and the slightly flattened character of the earth at the poles all change the earth's attraction for neighboring bodies, including satellites. Satellites wobble as this attraction varies. And the effect can be observed. The job of mathematicians is to describe the cause of the wobbles in an equation.

The Skill-Centered General Science Course

David E. Newton

Ottawa Hills High School, Grand Rapids, Michigan

The most significant dichotomy in science education today is our two-faced position on the teaching of scientific attitudes and skills in the modern science curriculum. We evidence great belief, ***in theory***, that the development of these attitudes and skills should be one of the primary goals of science teaching. Yet, in actual practice, this aspect of science is more often than not relegated to a secondary role. Current textbooks, teachers' guides, course outlines, and syllabi all contain extensive lists of scientific attitudes and skills, and considerable emphasis is placed upon the basic role of these in the teaching of the course. Teachers are encouraged to make wide use of the lists in their everyday teaching. However, when one extends his observation to the essential and basic structure of the textbooks and of actual classes, one discovers an almost fanatical concern for the subject-matter approach; i.e., for facts. Attitudes and skills are buried under the weight of the page upon page of pure factual information which characterizes these resource materials. The best indication of the fact-centered curriculum which currently engulfs us is the testing program used in our classes. How many national standardized tests or those accompanying textbooks, and how many individual teacher-prepared tests actually test for growth in attitudes? The number which do must certainly be very small.

This subject-matter approach is very desirable in the specialized science areas of the upper grades—biology, chemistry, and physics. These courses are normally geared to the college preparatory students who must digest certain basic facts and develop certain specialized skills for their succeeding courses in science. However, in the general science courses which are typically taught in the junior high grades, this justification of fact-emphasis is not valid. In the first place, we can not justify junior high general science as a college preparatory course to the same degree as biology, chemistry, and physics. This course occurs too early in the student's academic career. Retention studies indicate how few facts students retain after one or two years, and more is forgotten by the time they reach college. Furthermore, in many areas general science is a required course keyed to slower students, and a college-preparatory justification is even more unreasonable.

In the second place, the increasing number of schools now offering science in the elementary grades creates a severe problem for the traditional junior high general science curriculum. The familiar compre-

hensive survey of the major sciences is now superfluous—indeed, interest killing—in many schools; the students have already been exposed to the descriptive approach in the grades.

One other rationalization of the present fact-centered general science curriculum is that students at any ability level should have some knowledge of scientific principles as they operate in our everyday lives. Knowledge of this type may be used to stimulate the student's interest in science, and, in such a case, we have no criticism of the argument. However, if this knowledge is intended to turn students into proficient do-it-yourself repairmen, doctors, and weather-forecasters—as some teachers have tried to do—we doubt the value of the approach. Modern technology is highly complex, and one can hardly expect to give the junior high student the background in electricity which will permit him to repair his own television set.

The search for a justification of junior high general science has resulted in the development of a proposed new curriculum for this area of study. The major change in the new approach would be an emphasis on scientific skills and attitudes in practice as well as in principle. At this point, it should be made clear that we are not so naive as to believe that teaching of any kind can occur without factual information. However, the way in which subject matter is used is of the utmost significance. Instead of organizing the course around units of factual information, such as: air, light, heat, water, etc. (which structure clearly indicates the points of emphasis), the basic units would be centered on scientific skills and attitudes, as: observation, description, experimentation, research, etc. Thus, instead of learning, in the first four weeks of a general science course, about the atmosphere—its parts, characteristics, uses, and other facts which are soon forgotten—one would study the methods of observation. Important scientific skills would be taught, in this way, *as* important skills; they would not arise as mere by-products of learning some factual information.

A description of the manner in which a skill-centered unit could be developed might be helpful at this point. Consider the unit on observation as an example. The introduction would be used to indicate to the class how poorly developed (relatively) is their sense of observation. A mock crisis, for example, may be staged by the teacher, planned to occur during the class time. Pupils are asked to describe in detail the persons involved in the event and the actions which occurred. When the need for more acute and accurate observation has been established in this way, the teacher turns to two or three simple exercises which have as their goal the development of this skill. The identification of minerals and rocks is an exercise admirably suited to

this purpose. Other kinds of classificatory exercises are also appropriate.

Once the students have had this opportunity to sharpen their natural senses, it is necessary for the teacher to point out the inadequacies of the human senses *per se*. An excellent means of doing this is by a study of optical illusions. Such a study points up the limitations and inaccuracies of human eyesight. This transition section leads naturally into a study of the mechanical devices intended to augment the human senses in observational work. Such devices include most obviously the microscope and the telescope. However, the seismograph, the Geiger counter, the spectroscope, and a number of other gadgets are just as logically placed in this category. A somewhat extended study of the structure and use of each of these instruments can give the class a feeling for the wide variety of aids which a scientist has at his disposal. The major method of instruction in this unit, and all others, is, of course, class participation. The main objectives of the skill-centered unit would be defeated if the instructor lectured or the students read on any subject about which they could learn by their own first-hand experiences. Thus, in the study of the microscope, it would be expected that the students actually learn how to use it and to make the instrument serve in developing their skills of observation. The great value of drawing what one sees is evident.

Units such as these, based on observational, descriptive, experimental, and research skills, are highly practical when the teacher approaches them with a high degree of imagination. Most important, such units bring the emphasis in general science teaching back where it belongs—on scientific skills and attitudes.

STANDARDS FOR TINY SCREWS TO EASE MISSILE REPAIR

The American Standards Association has written standards for screw threads so fine that they cannot be seen with the naked eye.

Lilliputian screws used in delicate instruments and controls for missiles and rockets are so tiny that the smallest can be hidden in the dot of a typewritten "i." About 75,300 of these are required to fill a thimble. Their diameters range in size from 0.01 inch—about three times the diameter of a human hair—to 0.06 inch.

No standards for such screws have been available until now. Thus in the absence of standard screws, even in the watch industry, it became necessary for engineers to design special screws for their new missile instruments and controls.

Not only was this a waste of time, but the mounting complexity of stocking spare screws for maintenance of the instruments was rapidly producing headaches around the country.

In time, said the ASA, it may be possible to limit production and inventories to several lengths of screw in each of 14 standard diameters. The standards were drawn up by an ASA committee with backing of the American Society of Mechanical Engineers and the Society of Automotive Engineers.

Teacher Load in Science and Mathematics

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The axiom that a good program in science and mathematics depends upon the quality of the teaching staff has been advocated by educators and others interested in education throughout history. Certainly, capable teachers enhance the quality of the educational program. Frequently, however, the tendency has been to overload capable teachers in science and mathematics to the detriment of the quality of instruction and to the overall efficiency of the teacher. As Chisholm has pointed out clearly:

> Unless the responsibility for an equitable assignment of duties among the teachers is kept in mind, it will be significantly violated . . . the net result is that those who are best equipped to contribute the greatest amount in carrying on an effective educational program in the school are so overburdened that their efforts often are forced to a level of actual mediocrity.[1]

Teacher load in science and mathematics is of paramount importance, then, in view of its effects on the quality of instruction and the welfare and morale of the teacher. The federal government, professional organizations, community groups, educators, and schools throughout the nation have been genuinely concerned about programs in the areas of science and mathematics. New courses have evolved; new approaches and contents for courses suggested; ideas for providing for all levels of ability proposed; emphasis given to creative teaching, providing additional experiences at all levels for pupils; teacher workshops in science and mathematics have been carried on; and a myriad of other activities and suggestions for schools have been outlined. In most of the proposals that have been made recently for science and mathematics areas, little or no thought has been directed to the work load of the teacher. Providing additional experiences for talented pupils in these areas, means that the teacher must give additional time to planning for learning, developing instructional materials, and adjusting to all levels of ability.

New courses and methods of instruction make it necessary that the teacher keep abreast of trends and proposals in the areas of science and mathematics. Teacher workshops and committees require extra time and effort on the part of all teachers involved. In short, science and mathematics teachers have been asked to improve instruction, provide for the optimum development of each pupil, initiate new courses and methods of instruction, and work with children and youth in out-of-class projects without much attention to additional load.

[1] Leslie L. Chisholm, *The Work of the Modern High School*, The Macmillan Co., New York, 1953, pp. 468–69.

Just what is the teacher load of teachers in mathematics and science? Can teacher load be objectively measured? What effect does heavy teacher load have on the improvement of instruction in mathematics and science? How do teachers of science and mathematics feel about present teacher loads? What suggestions do teachers of science and mathematics have for improving teacher load situations? What recommendations and/or considerations should be studied when initiating new innovations in science and mathematics concerning the load of the teacher? The remainder of this article will attempt to present answers to these important questions.

Several major research studies have been made to determine objectively the actual load of teachers in the areas of science and mathematics. The Douglass formula[2] for measuring teacher load has been utilized in numerous research investigations since this formula appears to be the most objective and comprehensive teacher load formula which now exists. Furthermore, the formula represents the work of Douglass and others for a period of over 32 years. The formula takes into account all of the important factors of teacher load such as subject and grade level, class periods taught per week, number of pupils, duplicate preparations, length of class periods, and cooperative duties of the teacher such as supervision of out-of-class activities, teachers' meetings, and administrative duties. The formula yields an index number for a result which tells the number of teacher load units the individual has per week. Douglass defines one teacher load unit as being equivalent to teaching and preparing for an average class of 25 pupils for one period of 50 minutes—ordinarily 84 minutes of work.[3] Douglass in 1954 presented the following indices as national norms for teachers in science and mathematics:[4]

	Lower Q.	*Md.*	*Upper Q.*
Mathematics	25.4	29.6	34.1
Science	25.4	30.4	34.0
All Subjects	27.3	29.9	32.9

Odell[5] utilized the Douglass formula to measure the load of teachers in Illinois in 1949. The indices for science and mathematics resulting from this study were as follows:

	Q_1	*Md.*	Q_3
Mathematics	27.3	30.2	32.3
Science	28.1	30.7	31.3
All Subjects	25.5	29.2	32.0

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[2] Harl R. Douglass, "The 1950 Revision of the Douglass High School Teaching Load Formula," *The Bulletin of the National Association of Secondary School Principals*, May, 1951, p. 14.

[3] Harl R. Douglass, "Applying the Revised Douglass Formula for Measuring Load of High School Teachers," *The Bulletin of the National Association of Secondary School Principals*, October, 1952, pp. 67–68.

[4] Harl R. Douglass, "Revised Norms for High School Teaching Load," *The Bulletin of the National Association of Secondary School Principals*, December, 1954, p. 98.

[5] C. W. Odell, "Teacher Load in Illinois," *The Bulletin of the National Association of Secondary School Principals*, January, 1949, p. 94.

In a major research study in 1958–59, it was found that the median loads for science and mathematics were close to the median for all subject areas. The study included 363 high school teachers in cities from 5,000 to 25,000 population. The indices for teacher load of teachers in this study by subject areas follow:[6]

TABLE 1

INDICES FOR TEACHER LOAD OF TEACHERS BY MAJOR SUBJECT AREA

Subject Area	Q_1	Median	Q_3
English	29.20	32.72	36.07
Art	24.95	29.12	33.18
Home Economics	28.85	30.81	34.01
Music	24.07	32.26	34.30
MATHEMATICS	28.09	29.80	31.89
Agriculture	34.09	35.63	42.92
Industrial Arts	25.05	28.27	32.85
Physical Education	27.52	30.68	34.68
Commerce	26.61	30.61	32.85
Social Studies	29.88	33.01	37.77
Foreign Language	25.94	29.35	32.54
SCIENCE	28.11	30.99	34.43
All Areas	28.07	30.98	34.68

In the same research investigation, it was found that the average teacher load for science and mathematics teachers was 32.17 and 30.17 respectively compared to the highest subject area average in agriculture of 37.94 and the lowest average in art of 29.54 units.

The effects of heavy teacher load are reflected in the following comments made by school administrators in a study on mathematics in 1955. These statements by administrators point out clearly the importance of teacher load on the quality of the programs offered in science and mathematics.

One school administrator comments:

I think that most educators will agree that many programs in mathematics are inadequate. Until classroom loads are made smaller or teacher load is decreased the problem will continue.

Other comments by administrators are as follows:

Problems in mathematics are increased by the shortage of room and teacher time. Any kind of flexibility is difficult in a very crowded situation.

I believe our problem is that of most small schools. Each teacher is over-worked. Our building is overcrowded in spite of the fact that it is relatively new, consequently we are barely able to offer the minimum in every field.

The problem of teacher load is a chief stumbling block in improvement in our programs in mathematics and science. New developments in these areas have

[6] Monte S. Norton, *Teacher Load in Nebraska High Schools in Cities from 5,000 to 25,000 Population*, Doctoral Dissertation, University of Nebraska, 1959, p. 130.

placed added loads on the shoulders of the teachers. It is not always possible to hire additional help, thus, additional effort and work become the burden of the teachers who already have more than enough to do.

In a recent study of teacher load, science and mathematics teachers were asked to estimate their present load in terms of being reasonable or light against being heavy or extremely heavy. Of the participating teachers, 38.9 per cent of the mathematics teachers and 42.1 per cent of the science teachers estimated their teacher load as heavy or extremely heavy. In regard to the enjoyment of their present teaching assignments, 25.0 per cent of the teachers liked their work "fairly well" or "not especially" while 75.0 per cent said that they were enjoying their work "very much" or "particularly enjoying it." Science teachers participating in the study stated they were enjoying their work "fairly well" or "not especially well" in 44.8 per cent of the cases. Another question directed to science and mathematics teachers was concerned with feeling of strain and tension in teaching assignments. Nearly 17 per cent of the mathematics teachers felt considerable strain and tension in teaching, while 31.6 per cent of the science teachers expressed considerable strain and tension in present teaching assignments.

In regard to the number of pupils in classes per week, science teachers on the average had 117 pupils per week while mathematics teachers had an average of 120 pupils per week.

The foregoing facts are important for many reasons. Attitudes and feelings toward the work load on the part of teachers weigh heavily upon the effectiveness of instruction. Present data concerning teacher loads of teachers in science and mathematics are important if intelligent planning and best solutions to problems in this area of education are to be brought about.

What suggestions do teachers of science and mathematics have for improving teacher load situations? A few suggestions by teachers in these areas follow:

(Mathematics, man teacher) I feel that a thorough study of teacher load should be made on a cooperative basis. Reasonable policies on what constitutes an average load should be determined. Some formula should be used or developed by the administrators that would objectively measure the load of each teacher. Adjustments need then be made in view of the findings.

(Mathematics, woman teacher) I feel that more clerical help is needed. Teachers should be relieved of the duties of recording grades, tests, and keeping counseling records for the office. This requires entirely too much of the teacher's time.

(Science, man teacher) Extra-curricular duties seem to be our greatest problem in teacher load.

(Science, man teacher) Group students on ability basis.

(Mathematics, man teacher) The only suggestion I would have would be to make sure each teacher has a full vacant period—in other words not to assign supervision during this hour.

(Science, man teacher) We need to do some intelligent thinking and study on this problem and present results to the board and the community.

In view of the foregoing information, it would seem advisable for administrators and others involved in the educational program in science and mathematics to consider carefully the following recommendations when assigning work loads to teachers. If a school desires an optimum program in the areas of science and mathematics, it must give serious attention to the paramount problem of teacher load. Some guidelines and considerations to be made in teacher load are:

1. All factors that constitute the total work of the science and mathematics teacher must be considered when determining assignments. Pupil-teacher ratio is only one of the teacher load factors involved. Attention must be given to number of preparations, type and grade level of the class, number of periods taught per week, cooperative duties, and other aspects of the total load of the teacher.
2. Teacher loads of teachers in any subject area can be measured objectively by the use of a formula such as the one developed by Douglass. Through objective measuring of teacher loads, equitable assignments for each teacher can be more nearly determined.
3. Changes in offerings, additional courses, more pupils in science and mathematics, teacher workshops, and other activities in the science and mathematics fields do not guarantee an improved program in these areas. Although such activities are necessary and important, primary consideration must be given to the teachers who are expected to carry on the educational program. Steps must be taken to insure the continued high quality of instruction by capable teachers by giving them time to plan and prepare for quality instruction as well as the necessary facilities and atmosphere most conducive to learning.
4. Extra curricular duties must be wisely assigned by administrators. In one study of teacher load it was found that teachers of science were spending nearly 25 per cent of their work week on cooperative duties not directly related to teaching. Mathematics teachers were spending nearly 22 per cent of the week on cooperatives.
5. Teachers need adequate help from principals and supervisors. Helpful suggestions from persons in administrative positions can do much to relieve the work load of the teacher. The most capable teachers in the school deserve their fair share of the supervisor's time. These persons who are in administrative positions are best equipped to obtain materials and other helps that will be of value in science and mathematics programs.

Efficient and effective programs in science and mathematics depend upon efficient and effective teachers in large part. The problem of teacher load in science and mathematics is indeed an important problem for the nation.

FORESEE CHEAP ROCKETS FOR DAILY WEATHER USE

A cheap plastic rocket that could be fired daily by Weather Bureau personnel in large cities, in order to provide more accurate weather forecasts, is foreseen as a development following the successful firings of the ARCAS rocket.

ARCAS stands for All purpose Rocket for Collecting Atmospheric Soundings. It is a solid fuel, low cost meteorological rocket that can be launched by a two-man crew.

Eventually the rocket will be made of finely spun glass fibers so that it may be fired over populated areas, then exploded into harmless fragments when the desired information has been gathered.

A Rationale for the Teaching of Biology

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A. The High School Biology Problem

The present high school biology offering is an outgrowth of earlier course types which have formed a part of secondary education since the days of the earliest American high schools more than a century ago. The curriculum of these schools included courses in natural history, which, although they were taught from the viewpoint of nature study and natural philosophy, occupied much the same position in the curriculum as the high school biology course of today. These gave way to separate botany and zoology courses, influenced by the introduction of laboratory methods at the college level by Louis Agassiz and Asa Gray in the 1870's. Such high school courses emphasized taxonomy, dissection, morphology and, to a lesser extent, physiology. Finally, in the 1920's and later, integrated general biology courses were introduced, and replaced the older kinds.[1]

Little has been done, however, in the forty years since the introduction of general biology courses in high schools, to evaluate the offering or arrive at a workable definition of its function. At present, there are three general kinds of biology courses offered at the high school level: (1) the "types" course, utilizing mainly the taxonomic approach; (2) the "principles" course, which attempts to integrate biological materials around the functioning and behavior of living things, with emphasis on the physiology of cells, organs and organ systems; and (3) the "consumers" course, which involves an attempted organization on the basis of the needs of students, real or presumed.

The first two kinds are mainly watered-down versions of freshman college courses, perpetuated largely by the tendency of high school teachers to follow the easy road of teaching biological materials as they have learned them in their college courses. This is in turn the result of failure on the part of college curriculum-makers to recognize the needs of prospective high school teachers in terms of the needs of their future students. As a consequence, prospective teachers are placed in the same courses as premedical students and those planning on going into biological research. In many institutions which lack a well-developed general education program, various non-biological groups, such as English, history and economics majors, are placed in the same beginning biology course as are prospective teachers, premedics and research majors. The beginning biology course involving

[1] Rosen, Sidney, "The Origins of High School Biology," School Science and Mathematics, LIX (June 1959), 473–489.

one of the first two listed approaches, or some combination of them, is presumed to be the answer for everybody, with little or no thought being given to the diverse needs of those taking it. It is this course, then, which is, consciously or otherwise, copied at the high school level.

The third listed approach, the "consumers" course, is an amorphous category not widely represented among college courses. It is supposed to "meet the needs" of high school students, especially the non-college-oriented and terminal ones. Perhaps the best that can be said for it is that it does show the result of thinking and original planning on the part of its instigators, and constitutes a serious attempt to get away from the unquestioned following of established patterns, regardless of the needs involved. It breaks down, however, in that there is a general lack of agreement as to what the needs of high school students are, and even as to how to determine them. The same may be said of "consumers" courses in the few cases where they have been tried at the general education college level.

Textbooks are generally written to fit one of the three listed kinds of courses, or a combination of two or even all three of them. Although each kind of course or textbook generally makes a bow toward one or both of the other kinds, such that it is difficult to find a course or book of "pure" type, the result is that there are widely differing bases for the presentation of high school biology. The situation is made more diverse by varying degrees of laboratory emphasis, depending usually on availability of equipment and room, and size of classes. A further reason for diversity lies in differences in extent and kind of teacher preparation, and the ability of the individual teacher to extend and repair his own background, utilize new techniques and materials, and devise new class experiences for his students.

The result of this wide variation is that it is not possible to find any dependable basis for judgment of students' knowledge and understanding at the beginning of a freshman college course in biology, following their completion of high school biology. Based upon thirty years' experience in the teaching of biology at the freshman-sophomore college level, the writer has come to the position of assuming ***no*** difference between freshman students who have had, and those who have not had high school biology, because, with rare exceptions, no difference can be detected after the first three weeks of the college course. Actually, those who have not had high school biology often rate higher at the end of the course than those who have had it. All of this is in the face of the fact that most high school biology courses are simply watered-down copies of college courses.

Mallinson, in a provocative article in *The American Biology*

Teacher,[2] has pointed out that while two-thirds of all high school students in the United States take biology, in a typical group of forty students, only four will ever take another course in biology, and of these only one will go beyond the introductory college course. He expresses the opinion that the present typical course in high school biology serves adequately neither the terminal student nor the college entrance student. It is inadequately adapted to the needs of the former of these two groups, and has little carry-over value for the latter, in that it is taught at a grade level when their intelligence is still immature from the standpoint of the subject matter which is included in it. Furthermore, he raises the questions of (1) whether much attention is paid to the problem of the functions which the high school biology course is expected to fulfill, other than meeting a laboratory science requirement for graduation, and (2) whether, even if consideration is given to the matter of function, the course is actually organized and taught in such a way that this is recognizable.

B. A Re-Thinking of General Biology

In the light of the problem which has been stated, it might appear that the answer would lie in an attempt to attain uniformity of content and presentation for one or more courses in biology at the high school level. It has been proposed[3] that one biology course for all students be offered at the 9th grade level, and an additional elective course for advanced students be offered at the 12th grade level. Others have suggested separate courses for terminal and college-entrance students (a two-track system).

It is the belief of the writer, however, that regardless of the level at which biology is taught, uniformity of subject matter and presentation, even if it were possible in so broad a field, is not necessarily desirable. What is necessary, rather, is to develop a common basis for understanding in all students, whether they are college-entrance students or not. In order to do this we need to re-think what "general biology" is from the teaching standpoint, and particularly what *ideas* we want our students to come out with.

This is not a problem of the high school alone. It is a problem common to the teaching of biological materials at all levels K–14 (kindergarten through junior college). All biology taught K–14 is general biology. At elementary levels (early, middle and later) it is integrated into elementary science. At the junior high school level it begins to emerge as a segment of general science. At the senior high

[2] Mallinson, George G., "Biology—An Anomaly," *The American Biology Teacher*, XX (November 1958), 248–250.

[3] Mallinson, *op. cit.*

school level it emerges full-blown as "general biology." It retains this status through the junior college level.

At the senior college and graduate levels, biology takes the form of specialized "area" courses which constitute tools for advanced study, professional study and research. These include courses in taxonomy, physiology, anatomy, histology, embryology, ecology, genetics, parasitology, entomology and other specialized sub-disciplines. They may be further sub-divided into plant and animal branches, branches related to certain taxonomic groups within these major categories, and elementary and advanced levels.

The need is to find a common basis of understandings that can be developed in all students (including high school), who are taught under the general biology approach. If this can be done, they will carry these understandings into out-of-school living, if they are terminal students, and/or into advanced, professional and graduate study, if they are so bound, using them in any case as a basis for integrating their later experiences.

Such common understandings must not be dependent on the selection of particular areas of subject matter, or particular methods of presentation. They must be in the form of *ideas* common to all biological subject matter by virtue of its biological nature. They must therefore be approachable and teachable through the medium of *any* reasonably large selection of biological subject matter, or in terms of any biological area. Furthermore, they must be approachable and teachable at *any* educational level where general biological material is used.

It may appear that the location of such ideas is too large an order; but if such is true, then perhaps it is time that we raised the question of whether there is such a thing as "general biology," or even whether biology is a unified science at all, rather than a family of related sciences. The writer believes that it is possible to locate such pervading ideas, and that in order to find them we have only to delve deeply enough into life science and ask ourselves what its unique characteristics are.[4]

What may constitute the basis for these common understandings which we can legitimately hope that all "general biology" students, at whatever level, will attain? One idea is primary: *Life is a matter of dynamic interrelationships, ever changing, never standing still, understandable only in terms of its totality as a constantly shifting picture.* This is the unique characteristic of life, and of biology as the science of life. It is that which differentiates life science clearly and unequivocally from its sister sciences.

[4] Van Deventer, W. C., "The Use of Subject Matter Principles and Generalizations in Teaching," School Science and Mathematics, LVI (June 1956), 466–474.

This idea furnishes an avenue through which all major biological areas can be successfully approached for teaching. This is simply another way of saying that it is a unifying fabric which runs through all of th m, or that it constitutes a major portion of the basis on which they rest. The following are brief outlines of four such areas which are commonly taught in general biology courses, indicating the subtopics under each where the pervading idea of dynamic interrelationships applies:

1. The balance of nature (including)
   a. Energy and chemical relationships within any plant-animal community
   b. Relationship of all plant-animal communities to changing physical environmental factors
   c. Operation of environmental gradients, thresholds and the law of the minimum
   d. Plant-animal-human relationships in the total world of life
2. Heredity (including)
   a. The necessary interaction of heredity and environment in the expression of any particular trait
   b. The gene as a "place where" rather than a "thing which" on a macromolecule of desoxyribonucleic acid (DNA)
   c. The gene as the initiator of a chain of enzyme actions culminating in a specific characteristic or process—the total organism as a "bundle of enzymes" at work
   d. The probable origin of life through biochemical evolution, and the development of the present autotroph-heterotroph balance in the world of life through accumulation of balanced "loss mutations."
3. Evolution (including)
   a. Variation and natural selection within any taxonomic group
   b. The continuous adjustment of all living things to the demands of a changing environment
   c. The interaction of challenge and response in the history of evolutionary development of any group
   d. The relationship of the evolution of man to the attainment of the upright posture, the freeing of the hands for use as grasping organs, and the development and use of tools.
4. The human body in health and disease (including)
   a. The body as a community of cells, tissues and organs
   b. Functional and parasitic diseases as responses to unbalances and invasions of the body community
   c. The relationship of faulty body functioning and inadequate body defenses to lethal and partial lethal genes, and consequently imperfect enzyme chains.
   d. Study of the body in terms of interrelated functional areas, rather than traditional organ systems.

This concept of dynamic interrelationships, while well-adapted to laboratory treatment, does not depend heavily on laboratory experience for its comprehension. A few well-planned and carefully timed laboratory or field experiences, preferably of the open-ended sort—possibly one such broad experience to each unit—are sufficient. The collection of laboratory experiences which has been prepared by the project sponsored jointly by the National Academy of Sciences and the National Research Council, centering at Michigan State Uni-

versity, is an excellent source for some of these.[5] Others might consist of modifications or adaptations of traditional laboratory experiences, or those which might be originated by any imaginative teacher to take advantage of local opportunities and conditions.[6]

Extensive equipment is not necessary for teaching a course of this type. What is necessary is the kind of thinking on the part of both teacher and students which is involved in problem-solving operations. Elaborate audio-visual and other classroom equipment may sometimes constitute an actual barrier to this kind of thinking, through the temptation which it presents to both teacher and students to rely on ready-made, cut-and-dried presentation of subject matter. While the kind of thinking desired is that which is involved in the better types of laboratory and field experiences, it is by no means limited to them. It can take place equally well in connection with discussions and library investigations.[7] The student must be trained to think, to analyze and synthesize, independently. This is the opposite of rote learning. It involves the use of facts, but as tools to arrive at understandings, rather than as ends in themselves.

The concept of dynamic interrelationships can be taught at any educational level, and by utilizing any, or almost any biological materials. It is only necessary to select materials which are meaningful to pupils (i.e., within their range of comprehension), and to use vocabulary which they either understand or can be led to understand. Schultz[8] demonstrated the effectiveness of this approach in teaching 2nd and 6th grade pupils, utilizing a unit involving the study of an aquatic plant-animal community.

The biological materials utilized by different teachers in corresponding courses at the same level (e.g., high school biology courses) do not necessarily need to be the same, or to be taught in the same way. All that is necessary is that the basic idea be "gotten across" in terms of whatever materials and methods are used. The idea of dynamic interrelationships is a kind of "least common denominator" for all biology.

It also makes no difference if the same materials (e.g., the biology of insects, change of leaf coloration in the fall, man in space, or what happens to food in the human digestive tract) are used at different

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[5] Lawson, Chester A., editor, *Laboratory and Field Studies in Biology: A Sourcebook for Secondary Schools*, Committee on Educational Policies, Division of Biology and Agriculture, National Academy of Sciences-National Research Council, Washington, D. C., 1957.

[6] Van Deventer, W. C. and Staff, *Laboratory Experiences in Biological Science*, Western Michigan University, 1959.

[7] Van Deventer, W. C., "Laboratory Teaching in College Basic Science Courses," *Science Education*, XXXVII (April 1953), 159–172.

[8] Schultz, Ida Beth, *A Way of Developing Children's Understanding of Ecology*. (Unpublished doctoral dissertation), University of Florida, 1955 (microfilm).

levels (i.e., repeated) in the pupil's experience. The idea of dynamic interrelationships is broad enough and deep enough that it bears repetition in terms of the same materials in a progressively sophisticated form at advancing levels.

Finally, while this approach to biology makes constant use of concepts native to the fields of physics and chemistry (e.g., the atomic-molecular theory of matter, the kinetic theory of heat, chemical bonds, catalytic and enzyme action), there are none of these that cannot be explained in simple and readily understandable terms for purposes of biological understanding. Physical and chemical knowledge in the form of prior courses taken is not a necessary prerequisite for a biology course of this type.

While physical and chemical tools and mathematical analyses must necessarily play an increasing role in certain avenues of biological research, the value of holistic, organismal and other typically biological approaches and methods of analysis must not be lost sight of, particularly at the general education level. Unless these broad understandings are built into the foundation of the future teacher, professional man or research worker, as well as the general citizen, a proper balancing and evaluation of the results of combined chemo-physico-biological research is impossible. Without this he "cannot see the forest for the trees."

C. Where Do We Go from Here?

Unless biology teachers of all levels get together and train themselves and their students of all ages in the basic understandings of biology and the nature of life phenomena, the results may well be tragic. Present emphasis on the teaching of science at all levels is principally in the area of the physical sciences. This is good and necessary, because this area had been badly neglected. If the time were ever to come, however, when the only approach to biology at the general level lay through the physical sciences, we would lose heavily in an understanding of the unique characteristics of the world of life and of ourselves as living organisms. To prevent this, as well as to increase the efficiency of our teaching and improve its end-product, we need a re-vamping of our approach to general biology.

Physical scientists have dealt with a similar problem in connection with presenting physics at the high school level. Their problem was occasioned by the fact that modern developments in physics have outrun the content and methodology of the traditional high school physics course. This is a related but somewhat less complicated situation than that which we have outlined for biology. The Physical Science Study Committee, which operated at Massachusetts Institute of

Technology, utilized the concepts involved in wave motion as an approach to the teaching of high school physics.[9] The laboratory equipment which they suggest can be largely locally-made or easily obtained, and is not particularly complex.

The chemists are conducting a similar study, attempting to integrate beginning chemistry teaching around the ideas involved in an understanding of the electro-chemical bond. Groups of mathematicians at the University of Illinois, University of Maryland and Yale University are re-studying mathematics curricula and re-building beginning mathematics courses around similar broad understandings. Some preliminary experimental work in the physical sciences and mathematics indicates that this teaching of broad concepts can be carried into the elementary levels with success.[10]

In the area of the biological sciences, a development is in progress which may prove equally helpful. The Biological Sciences Curriculum Study, under the directorship of Dr. Arnold B. Grobman of the University of Colorado, and under the sponsorship of the American Institute of Biological Sciences, financed by the National Science Foundation, is carrying on a study of offerings in the life sciences at all educational levels (elementary, secondary, college and professional) and is considering problems of coordination among them. This project is described in detail in the April, 1959, issue of the *A.I.B.S. Bulletin*, in an article by Dr. Grobman.[11]

As a part of this study "a special committee will attempt to answer the general question, what should a student graduating from high school know about the biological sciences? The committee will attempt to determine what knowledge a citizen should have who has completed twelve years in our public schools. In conjunction with these studies it will be necessary to investigate the proper sequential arrangement of the new biology in the high school in line with other sciences and mathematics. It will also be an important concern to determine whether additional courses, perhaps at the twelfth grade, would be desirable for superior students, college preparatory students and other special categories of students."[12]

From this study could come the basis for a solution to our problem. It should be remembered, however, that no one person's or one committee's answer can be another person's panacea. Any program, no matter how well-planned, can degenerate into a deadly and monotonous repetition of factual learnings in the classroom and even in the

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[9] Little, Elbert P., Friedman, Francis L., Zacharias, Jerrold R., and Finlay, Gilbert, "The Physical Science Study," *The Science Teacher*, XXIV (November 1957), 316–330.

[10] Atkin, J. Myron, *An Analysis of the Development of Elementary School Children in Certain Selected Aspects of Problem-Solving Ability* (Unpublished doctoral dissertation), New York University, 1956.

[11] Grobman, Arnold B., "The Biological Sciences Curriculum Study," *A.I.B.S. Bulletin*, IX (April 1959), 21–23.

[12] Grobman, *op. cit.*

laboratory. The best-conceived program will break down unless the individual teacher is trained to be alive and concerned with the use of facts as tools for getting at understandings, which in turn become the inspiration for the acquisition of additional facts to serve as tools to attain further understandings. Russian science education, so frequently alluded to in this country by critics of our own science teaching, has been criticized recently by a Russian educational writer for exactly this kind of shortcoming.[13]

It is necessary to point out, furthermore, that there is a danger inherent in any over-all biology program which aspires to general or uniform applicability. The peculiar genius of biology as a teaching discipline lies partly in its diversity. This makes for adaptability to the special needs of individuals and groups, and to local teaching situations which frequently offer unique and valuable opportunities for learning important principles and relationships. Too much uniformity is deadening. The behavior of a falling object in a physics laboratory, or of a particular acid and base when combined in a chemistry laboratory is the same no matter where you observe it, but the behavior of a crayfish of a particular species in a stream in Illinois may differ from that of one of the same species in a stream in Michigan, because of the operation of environmental factors, few if any of which we can control, many of which we do not completely understand, and some of which we probably are not even aware of.[14]

It would apparently be better in planning a biology program, possibly in contrast to one in the physical sciences, to agree upon a limited number of basic understandings, and then to allow for the working out of the specifics of subject matter and methodology in any particular course at any particular level within the broad framework thus provided. In the 13th and 14th grade biology course of which the writer is chairman, the staff has agreed upon a set of broad principles and ideas to be included, but within this framework each teacher is free in a very large measure to vary subject matter and method. This makes for a maximum of originality and initiative on the part of the individual teacher, and allows for a maximum of adaptability of the course to the needs of particular groups of students.[15] A similar curricular philosophy could be made to apply to all teaching of biological materials at all levels.

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[13] Evronin, G. P. (translated by Ivan D. London), "On the State of Physics Teaching in the Russian Republic," *Science Education*, XLIII (April 1959), 270–274.

[14] Van Deventer, W. C., "Studies on the Biology of the Crayfish, *Cambarus propinquus* Girard," *Illinois Biological Monographs*, XXXIV (August 13, 1937), No. 100.

[15] Van Deventer, W. C. and Staff, *Basic Ideas, Generalizations and Subject Matter Principles Included in the Biological Science Course*, Western Michigan University, 1959.

Square-Off at Squares and Cubes

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James R. Newman in his "Commentary on Idiot Savants," *The World of Mathematics*, reports that when the Rev. H. W. Adams ordered ten-year old Truman Henry Safford to multiply in his head 365,365,365,365,365,365 by the same number, " 'He flew around the room like a top, pulled his pantaloons over the tops of his boots, bit his hands, rolled his eyes in their sockets, sometimes smiling and talking, and then seeming to be in an agony, until, in not more than one minute, said he, 133,491,850,208,566,925,016,658,299,941,583,255!' "

The methods which I suggest in this article for the rapid squaring and cubing of integers, I hope, will not prove so strenuous. But you, when you get to your units on exponents, can get your students to sit up and take notice. Instead of referring your students to the back of the book, have them make and check their own tables of squares and cubes. Although much of the work can be done mentally, an adding machine will be helpful.

Calculating a Table of Squares

First, furnish each student with a duplicated sheet showing the squares, the first differences (d_1), and the second differences (d_2), of the first five or six integers (see the tables in this article). Now, from memory, have each student fill in on the sheet the squares of the remaining integers up to and including 10. Now the fun starts, for a general rule, useful for rapid written or mental calculation, may be laid down for the squaring of any whole number:

1. Substitute a zero for the units digit and square the result.
2. Multiply the original units digit times double the tens digit with zero annexed.
3. Square the original units digit.
4. Add together all the results.

Example 1: Square 11.

1. Drop the units digit from 11 and substitute zero. This gives 10, which when squared, gives 100.
2. The original units digit, 1, multiplied by 20 (double 10), gives 20.
3. The original units digit squared, 1, is 1.
4. The sum of the results of the above steps is $100+20+1$, or 121, answer.

Example 2: Square 89.

1. 89−9=80. 80 squared=6400.
2. 9 times 160=1440.
3. 9 squared=81.
4. 6400+1440+81=7921, answer.

For finding the squares of other integers, follow similar steps.

Roots of Perfect Squares

By now you may suspect that perhaps square roots may also be determined rapidly—and so they can. Remember that in any table of squares, all the right-hand digits (excepting squares of 10 and multiples of 10) follow the symmetrical and easy-to-remember series: 1, 4, 9, 6, 5, 6, 9, 4, 1, Making use of this fact, it is possible to calculate almost instantaneously, the root, when perfect (i.e., integral), of any square.

Example 3: What is the square root of 6889?

1. In series, 9, the right-hand digit, falls 3rd or 7th. Hence the right-hand digit of the root is either 3 or 7.

2. 80 squared, 6400, is easily seen to be less than 6889; 70 squared would be too little, and 90 squared too much. Hence 83 or 87 is the required root.

3. Test the 3 and 7 in this manner: take 80, the square root of 6400, and multiply separately by each number, 3 and 7: 80 times 3, and 80 times 7. If one of the products, added to 6400, exceeds 6889, reject it; if neither exceeds, choose the digit used to obtain the product *closest* to 6889. In this instance the desired root is seen to be 83, answer.

Squares ending in the digit 6 are an exception. In testing for 4 or 6, first double the number being multiplied: for the square root of 1936, 40 squared=1600, so double 40 to 80; then multiply and accept or reject as before. The number 5 occurs only once in series and so it presents no problem.

Another method of determining roots is shown below:

Example 4: What is the square root of 7921?

1. Find by experiment the largest two-digit number with a zero in the units position, which, when squared, will still be less than 7921. In this case 80 is the number required, as 80 squared=6400, which is still less than 7921.

2. Subtract the square obtained from the perfect square: 7921−6400=1521.

3. Note that 1521 ends in the units digit 1. Accordingly, the square of either 1 or 9, which each likewise ends in 1, must be subtracted.

Whether to subtract 1 squared or 9 squared must be decided by experiment. Try 9 squared. Thus $1521-81=1440$.

4. Note that in step 1) above, 80 was the number required. Double this number; this gives 160. Now divide 160 into 1440. $1440/160=9$. Hence $80+9$, or 89, is the square root of 7921.

The square roots of other *perfect* squares may be obtained in similar manner.

CHECKING THE TABLE OF SQUARES

When the students have constructed their tables of squares up to whatever integer is desired, the results may be checked, using the process of differencing, in this fashion:

Referring to the duplicated sheets given them, the students should note the first few differences filled-in, for illustrative purposes, by the instructor. Thus they will see that:

n	n^2	d_1	d_2
1	1		
		3	
2	4		2
		5	
3	9		2
		7	
4	16		2
		9	
5	25		

Using this as a base, the students should calculate the squares of all succeeding numbers, as: $25+9+2=36$. Hence, the next square is 36, the next d_1 is 11, and the next d_2 is again 2. Therefore, $36+11+2=49$, the next square, and so on. These results may be compared with those previously obtained.

CALCULATING A TABLE OF CUBES

First, on a sheet of paper, have each student write, from memory, the cubes of the integers 1 through 10. Now here we go with the first example; the general method is:

1. Cube the units digit.

2. Add the product to the cube of ten times the tens digit.

3a. Multiply the units digit times the tens digit.

3b. Multiply the product obtained in step 3a) above by the original number.

3c. Multiply the product obtained in step 3b) above by 30.

4. Find the sum of steps 2) and 3c).

Example 1: Cube 89.

1. 9 cubed = 729.
2. 729 + 8 times 10, cubed = 512,729.

3a. 8 times 9 = 72.

3b. 72 times 89 = 6408.

3c. 6408 times 30 = 192,240.

4. 512,729 + 192,240 = 704,969, answer.

Another method is to:

1. Drop the units digit.
2. Cube the difference.
3. Triple the cube root and multiply by the cube root, and then multiply by the units digit dropped.
4. Take a fraction of the result obtained in step 3) above and multiply by the units digit dropped. The fraction will be 1/10 for integers 10 to 19, 1/20 for integers 20 to 29, . . . , 1/90 for integers 90 to 99, etc.
5. Add the cube of the units digit dropped.
6. Add together the results obtained in steps 2) to 5) above.

Example 2: Cube 89.

1. 89 − 9 = 80.
2. 80 cubed = 512,000.
3. 80 times 3 = 240; 240 times 80 = 19,200; 19,200 times 9 = 172,800.
4. 172,800 times 1/80 = 2160; 2160 times 9 = 19,440.
5. 9 cubed = 729.
6. 512,000 + 172,800 + 19,440 + 729 = 704,969, answer.

Roots of Perfect Cubes

To obtain cube roots, merely reverse the process of involution:

Example 3: What is the cube root of 21,952?

1. The last digit of the root is obviously 8. [In any table of cubes, all the right-hand digits (excepting cubes of 10 and multiples of 10) follow the series, 1, 8, 7, 4, 5, 6, 3, 2, 9.]
2. 20 cubed = 8000, but 30 cubed = 27,000. Hence 28 must be the cube root of 21,952.

Another way of proceding is:

Example 4: What is the cube root of 704,969?

1. Find, by experiment, the largest two-digit number with a zero in the units position, which, when cubed, will still be less than 704,969. In this case 80 is the number required.

2. Subtract the cube obtained from the perfect cube: 704,969 − 512,000 = 192,969.

3. Note that 192,969 ends in the units digit 9. Now recall that the cube of only one units digit ends in 9, as 9 cubed = 729. Hence 729 must be subtracted from 192,969. This leaves a remainder of 192,240.

4. Since 80 was the number required in step 1) above, triple this number, and multiply by the units digit 9 (noted as the ending digit of 192,969 in step 3) above). This gives 80 times 3 times 9 or 2160.

5. Divide 192,240 by 2160. The quotient is seen to be 89, the cube root of 704,969.

The cube roots of similar *perfect* cubes may be obtained in similar manner.

Checking a Table of Cubes

Assume that each of your students has calculated a table of cubes up to 100. Now how do they know whether or not their work is right? Write on the blackboard:

n	n^3	d_1	d_2	d_3
1	1			
		7		
2	8		12	
		19		6
3	27		18	
		37		6
4	64		24	
		61		6
5	125		30	
		91		
6	216			

Using this as a base, the students should be able to easily check their previous work, as: 216+91+30+6=343. Thus the next cube is 343, and so on.

A single period of classroom time is usually sufficient to explain the procedures for either squares or cubes, but of course students should be given additional time, perhaps as a homework assignment, to work-out their tables. Students for once will realize, with confidence, that it was they that made and checked their own tables—not once did they have to rely on the work of someone else!

Bacteriophage in the Classroom

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As a particle which invades a bacterial host and multiplies several hundredfold leaving death in its wake, bacteriophage might well be termed "microbe hunter extraordinary." It was in 1915 that Twort first observed the tell-tale clear areas or plaques in Staphylococci cultures he was studying. On drawing his needle, with a bit of plaque, across the face of the culture, he soon noted dissolution as the track became transparent within a few hours. Two years later d'Herelle found similar bacteriolytic agents in sewage and stools and named them *bacteriophage*.

At first, hope ran high that here, indeed, was a common solution to the many ailments to which man is heir. Bacteriophage was employed therapeutically against the microorganisms causing cystitis, bacillary dysentary, plague and cholera. In the main, however, the results were far from promising, for it appears that 'phage (as it is known to many working with it) stubbornly refuses to act in the presence of body fluids and feces.

But to the well-informed biology teacher 'phage offers some relief for at least one of his ills: the inability to demonstrate "live" the presence or activity of an ultramicroscopic organism like a virus. Some natural reluctance stems from the fact that viruses are considered too "hot" to handle; almost all are pathogenic and require complicted techniques to demonstrate. Moreover, viruses cannot readily be cultivated in the classroom, nor can they be counted or their effects observed in a reasonable time or manner. Bacteriophage, although it is in reality a bacterial virus, suffers from few of these drawbacks. Its use presents a practical means whereby any or all of the following may be shown dramatically, and in the classroom:

—An invisible particle hovering between the living and the non-living,
—Cytolytic action on bacteria,
—Development of resistant strains of bacteria, and
—Effective of physical and chemical agents on 'phage action.

A few simple techniques is all one need acquire for the demonstrations; possibly the biology teacher may desire to present the exercises as one or more laboratory lessons. In any case, the materials are easily come by: Escherichia coli, Type B,† may be re-

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† The specific strain, E. coli, Type B, *must* be used in these experiments. Other varieties of coliform bacteria obtainable from feces and soil are completely unsuitable for the "T" series of 'phages recommended here. In addition, the other bacteria may be pathogenic while E. coli, Type B, is non-pathogenic. It is well to remember that states like Michigan require by law that a school must register with the State Health Department before experimenting with pathogenic organisms.

ceived from the Carnegie Institution of Washington, 1530 P Street, N.W., Washington, D. C. or from a neighboring microbiological laboratory; sterile, disposable petri dishes are purchasable from Scientific Products, American Hospital Supply Company, Evanston, Ill.; and various strains and titers of bacteriophage and their hosts may be obtained from the American Type Culture Collection whose address is: 2112 M Street, N.W., Washington, D. C.

The following basic techniques are directed primarily to the teacher:

Culturing Bacteriophage

Because 'phage is offered by suppliers in lots of a few milliliters it is advisable to culture it immediately to obtain reasonable qualities for demonstration and laboratory work. And it so happens that while it is being cultured, cytolysis may also be shown.

Simply prepare 50 milliliters of sterile nutrient broth in an Ehrlenmeyer flask, add five milliliters of E. coli, Type B, suspension and incubate until the broth becomes turbid, which usually requires from 15 to 30 minutes. Then pipette up to five milliliters of 'phage into the flask, incubate and examine every 15 minutes. Sudden clearing of the broth indicates cytolysis of the bacteria and the presence of very large numbers of 'phage particles—usually from 10^7 to 10^9 per milliliter. The resulting high titer or concentration of 'phage is extremely valuable in the experiments that follow.

Should the suspension fail to clear completely, the excess bacterial cells may be removed by shaking with a few drops of chloroform and centrifuging in sterile tubes. The supernatant which contains the 'phage may be poured off and stored for months in a refrigerator.

Demonstrating 'Phage Action

Twort's method may possibly provide the simplest means of showing 'phage at work in the classroom. In essence, the procedure is one of streaking high titer 'phage over a seeded nutrient agar plate; a technique which is also readily adaptable for project and problem solving activities.

Start by melting a sterile nutrient agar butt (test tube half-full of rehydrated nutrient agar) in a water bath. Then cool the agar to 45–48 degrees C., add 0.5–1.0 milliliters of E. coli, Type B, suspension (known as seeding), pour into a sterile petri dish and allow the seeded agar to harden. Using a sterile transfer needle, streak high titer 'phage on the surface of the hardened agar seeded with E. coli, Type B. (See Figure 1.) Incubation for 24 hours should reveal a clear area along the line of streak as well as circular plaques or 'phage colonies at the end if the streak has covered a large enough area.

FIG. 1. Streaking phage.

To determine at which temperature 'phage is inactivated, place a test tube of 'phage in a water bath and heat slowly, noting the increasing temperatures on a thermometer held in the water. At intervals of 10 or 20 degrees, Centigrade, beginning at 20 and proceeding to about 90 degrees, remove loopfuls of 'phage with a sterile transfer needle and make parallel streaks on a previously prepared seeded agar plate. The point at which 24-hour incubation no longer produces a clear streak is the inactivation temperature. (See Figure 2.)

Problems involving other physical and chemical factors may be approached in a manner similar to that employed with temperature. For example, one might investigate such problems as: At what pH is 'phage inactivated? Will ultraviolet light inactivate bacteriophage?

Developing Resistant Strains

If an entire seeded agar surface is streaked with high titer 'phage, 24-hour incubation will produce a perfectly clear plate. But what will

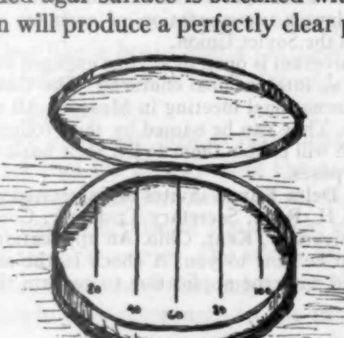

FIG. 2. Testing temperature.

happen after 48 or 72 hours of incubation? To find out, one need but swab completely the surface of a seeded plate and incubate for three days. Pin-point colonies in a matrix of clear nutrient agar may denote one resistant strain of Escherichia coli, Type B. Is this a mutant? Is bacteriophage resistance inherited?

From experience, the writer is convinced of the great value in inviting bacteriophage into the classroom. For, once the above techniques have been mastered, the teacher or student will be presented with a wealth of ideas, ideas translatable into demonstrations, laboratory exercises, projects and problems. Is an invited guest with these gifts not welcome?

THE COMPARATIVE EDUCATION SOCIETY
AND THE
COMMISSION ON INTERNATIONAL EDUCATION OF
PHI DELTA KAPPA
ANNOUNCE THE
COMPARATIVE EDUCATION SEMINAR AND
FIELD STUDY FOR 1960

"The Big Reforms in Soviet Education"

The Trade Union of Educational and Scientific Workers of the USSR has once again invited the Comparative Education Society to participate in a field study and seminar planned and directed by the Trade Union for American educators. In 1958, seventy-one professors participated in a five-week series of conferences throughout the Union. This was not a tour. Rather it was an intensive firsthand study of Soviet education. In 1960, however, the emphasis will be upon the changes which have been introduced as a result of the big reforms of 1959–60. The dates have been tentatively set for August 14 to September 17.

The final cost of participation in the program has not yet been fixed but it is expected that it will be about $1700. This will include all expenses within the Soviet Union, trans-atlantic transportation of economy class but tourist in Europe and first-class in the Soviet Union.

Participation in the program is open to anyone engaged in the college teaching or educational work of an international character. The Comparative Education Society will hold its international meeting in Moscow. All who go must become members of the Society. They can be named by their college presidents as institutional delegates which will enable them to list their basic expenses as a deduction for income tax purposes.

The Society and Phi Delta Kappa invites all interested persons to send their inquiries to Dr. Gerald H. Read, Secretary-Treasurer, Comparative Education Society, Kent State University, Kent, Ohio. An application form for participation in the seminar will be sent to you. A check in the amount of $50.00 per person must be returned with the application to confirm the reservation.

Modern Applications of Exponential and Logarithmic Functions

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Two Basic Aspects of Instruction in Modern Mathematics

The modern program of secondary-school instruction in mathematics includes among its basic fundamentals the following two considerations:

1. A strong emphasis on *functions*.

For example, the central concern in the modern study of logarithms is not the obsolete use of logarithms as a computational tool in abbreviating the labor involved in the solution of triangles. Instead, attention is centered about the study of the logarithmic *function*. Thus, *formulas* in which the logarithmic function appears may be especially helpful to the mathematics teacher with the modern orientation. The article below illustrates such formulas.

2. A strong interest in the applications of mathematics not only to the natural sciences but also to the *social* sciences.

For example, one finds in modern mathematical literature applications of mathematics to such sociological phenomena as the spread of rumor and prejudice and the state of marriage in primitive societies. The article below provides illustrations for mathematics teachers of the *widening* range of applications of modern mathematics.

The Objective of This Article

Now, according to the lyrics of a recent popular song, love and marriage go together like a horse and a carriage. But the mathematician is hardly satisfied with such a statement. He wishes to know *precisely* how they "go together."

In this article, we shall see how certain mathematical functions provide a wonderful tool for *illuminating* the analysis of the growth of love, the growth of population, and the relationship between love and population. A basic purpose of the article is to indicate how, in senior-high-school algebra courses, the study of the exponential and the logarithmic functions may be enhanced (i.e., motivated and made more meaningful) by references to the modern applications of these functions not only to the natural sciences but also to such social fields as sociology and psychology.

It should be realized that the mathematical exploration of such

social phenomena may be attacked by mathematical "weapons" *other* than the logarithmic and exponential functions explored here. Such "weapons" include the method of differences, the geometric mean, probability and statistical theory, set theory, etc.

Some applications below are recommended by the Commission on Mathematics of the College Entrance Examination Board for the mathematics curriculum of grade 12, and, they may be spelled out in greater detail via student construction of tables and graphs. Other applications may provide teachers with personal insight and perspective.

Population and the Exponential Function

The function $y=10x+5$ is an example of one of the simplest functions—the linear *algebraic* function. The function $y=10^x$ is an example of a special type of *transcendental* function—the *exponential* function. In both examples, y is a function of x. But in the second example the independent variable x is a variable *exponent*. We shall be concerned in this section with exponential functions.

Another example of an exponential function is the function $y=e^x$. (It may be recalled that, among other uses, the transcendental number e is employed in modern mathematics as a possible base for logarithms.) Both 10 and e are constants. Thus, the exponential function $y=b^x$ may include both examples of exponential functions, since b may equal 10 or e or some other constant.

A more *general* form of the exponential function is the function $y=ab^{kx}$. Here, as before, the two basic variables are y and x, while a, b, and k are constants. In the case of $y=10^x$, it may be observed that $a=1$, $b=10$, and $k=1$. In the case of $y=5e^{-x}$, it may be seen that $a=5$, $b=e$, and $k=-1$.

An *even more* general form of the exponential function is the form $y=ab^u$, where u is any function (algebraic or transcendental) of x. Thus, u, if *algebraic*, could be a polynomial function such as x^2+5x+6 or a rational function such as $(x-3)/(x-4)$ or an irrational function such as $(x-3)^{\frac{1}{2}}$. If u were a *transcendental* function, it could, among many possibilities, be a trigonometric function such as $\sin x$. In such composite or compound functions, y is a function of a function u. The basic variables are y and u (the auxiliary variable u being a function of x), and a and b are constants. In the case of $y=5e^{x^2}$, it may be seen that $a=5$, $b=e$, and $u=x^2$.

Now, the study of the exponential function is extremely important for at least two, not unrelated, reasons. First, the exponential function is widely used in *higher* mathematics. For example, it is a very common solution of elementary differential equations. Secondly, a tremendous number of uses of the exponential function have been

found in our *modern* age. Let us consider now some of these uses of the exponential function.

Thus, consider the study of the exponential function $y=e^{-x^2}$, whose graph is the famous standard *normal* probability curve. Such a function helps in opening the "door" to mathematics students interested in the role of chance in the modern statistical investigations of problems in both the natural and social sciences. Consider also the fact that *combinations* of the exponential function with other types of functions are frequently useful. For example, the function $y=ae^{-bx}\sin cx$ (whose graph, for $b>0$, is the damped vibration curve) has been applied to the study of problems in such fields as radio, electricity, the pendulum, and music.

Consider, too, the fact that the exponential functions $y_1=e^x$ and $y_2=e^{-x}$ may be "averaged" to yield a *new* function $y=\frac{1}{2}(e^x+e^{-x})$. Now, by definition, $\cosh x=\frac{1}{2}(e^x+e^{-x})$, and the graph of this hyperbolic cosine function is the *catenary*. Some insight into the utility of exponential functions may be obtained from the knowledge that the hyperbolic cosine function is used as a basis for correction for the sag of the engineer's steel tape. Furthermore, when inverted, the catenary provides the shape of a cross section of the dome of the Cathedral of Florence. Since it is possible to define the hyperbolic functions and even the ordinary circular (trigonometric) functions in terms of *exponential* functions, the mathematical "power and the glory" of the exponential function may be seen more vividly.

But even simpler forms of exponential functions have wide utility. Thus, the form $y=ae^{kx}$ has been applied in the analysis of the *growth* and *decay* of various types of "populations"—human beings, bacteria, trees, sugar, active yeast ferment, radioactive metals, electric currents, light, money, machinery-valuation, wounds, etc.

For example, the growth of a population in a community may follow the law $P=pe^{kt}$ where: p = the size of the *original* population at the time $t=0$, and P = the size of the population at some time t (in years). Thus, if, in 1955, the population of the community was 10,000, and in 1956 the population increased to 20,000, then the population *predicted* for 1960 may be computed in accordance with its law of growth as follows:

Part 1: Computing e^k	*Part 2:* Computing P
$P=pe^{kt}$	$P=p(e^k)^t$
$20{,}000=10{,}000\ e^{k\cdot 1}$	$=10{,}000\ (2)^5$
$e^k=20{,}000/10{,}000$	$=10{,}000\ (32)$
$e^k=2$	$=320{,}000$

Basic illustrations of the use of the exponential function in the analysis of the growth and decay of a "population" in such fields as *economics* and the *natural sciences* might include the following:

1. $A=1{,}000\ (1.03)^t$ where A = the "growing" amount of money being accumulated after t years of time, during which the principal of $1,000 has been invested, interest being compounded at 3% annually.
2. $P=1{,}000\ (1.03)^{-t}$ where P = the "decaying" present value of an amount of $1,000 due at the end of t years from now, interest being compounded at 3% annually. (In other words, P is the principal which must be invested now at 3% interest—compounded annually—so that the amount at the end of t years will be $1,000.)
3. $R=100\ (.95)^t$ where R = the "decaying" number of units of radium left after t units of time of decomposition of the original 100 units of radium.

   Note: $(.95)^t=(100/95)^{-t}$.
4. $I=20\ e^{-80t}$ where I = the number of units of electric current (in a given circuit) left after t units of time of dying of the original 20 units of current.
5. $C=0.00436\ e^{0.0253\,T}$ where:

   C = the electrical conductivity of glass, and

   T = the temperature (Farenheit).
6. $D=D_0e^{-kt}$ where:

   D = the difference between the temperature of a body (such as water) being cooled (in accordance with Newton's law of cooling) and the temperature of the medium (such as air) surrounding the body, and

   D_0 = the value of D when the time $t=0$.
7. $W=w\ (.95)^t$ where:

   W = the weight of an animal after t days of fasting, and

   w = the value of W when the time $t=0$.
8. $P=pe^{-kh}$ where:

   P = the atmospheric pressure (in pounds per square inch),

   h = the height above sea level (in feet), and

   p = the value of P when $h=0$.
9. $I=ae^{-bd}$ where:

   I = the intensity of light (the amount falling on a unit of surface) at any position in an absorbing medium (as water),

   a = the original intensity of light at the surface, and

   d = the depth of observation.

Love and the Logarithmic Function

Not *all* transcendental functions are exponential functions. Thus, the trigonometric function $y=\sin x$ is a function which is *not* al-

gebraic and which, therefore, is an example of a transcendental function. Other types of transcendental functions include the inverse trigonometric functions (such as $y=\text{arc tan } x$), the hyperbolic functions (such as $y=\cosh x$), and the inverse hyperbolic functions (such as ($y=\sinh^{-1} x$). We shall emphasize in this section those transcendental functions known as ***logarithmic*** functions.

The function $y=\log_{10} x$ is a simple example of the logarithmic function utilizing the "common" base 10. The function $y=\log_e x$ is a simple example of the logarithmic function utilizing the "natural" base e. Both of these examples of logarithmic functions are included in the general logarithmic function $y=\log_b x$, since b may equal 10 or e or some other constant.

A more ***general*** form of the logarithmic function is the function $y=a \log_b kx$. Here, as before, the two basic variables are y and x, while a, b, and k are constants. In the case of $y=\log_{10} x$, it may be observed that $a=1$, $b=10$, and $k=1$. In the case of $y=7 \log_e 2x$, it may be seen that $a=7$, $b=e$, and $k=2$.

A ***much more*** general form of the logarithmic function is the function $y=a \log_b u$, where u is any function of x. Thus, u could be a polynomial function such as x^2-4 or the trigonometric function cos x. In such composite functions, the two basic variables are y and u (the auxiliary variable u, again, being a function of x), and the constants are a and b. In the case of the composite function $y=\log_{10}$ $\sin x$ (for which tables of values are commonly found in elementary trigonometry books), it may be seen that $a=1$, $b=10$, and $u=\sin x$.

Like the exponential function, the logarithmic function is widely used in higher mathematics and is extremely useful in the mathematical analysis of twentieth-century problems. Thus, the inverse hyperbolic functions are easily defined in terms of logarithmic functions. Specific illustrations of the use of logarithmic functions in the fields of the ***natural sciences*** are given below:

1. $y=ax^2 \log_e (1/x)$ where:
   $y=$ the speed of signalling in a submarine telegraphic cable, a is a constant, and
   $x=r/t$ where
   $r=$ the radius of the core and
   $t=$ the thickness of the covering of the cable.
2. $\log_e p=(a/T)+b \log_e T+c$ where:
   $p=$ the vapor pressure of a given substance,
   $T=$ the absolute temperature, and a, b, and c are constants.
3. $y=\frac{1}{2}(\frac{1}{2}ax^2-\log_e x)\ a+b$. This is the equation of the pursuit curve of a fighter plane pursuing a bomber plane flying a course in a straight line, the speeds of both planes being equal and constant, and the nose of the fighter plane always being pointed at the bomber plane.

But even simpler forms of the logarithmic function have wide utility. Specific illustrations of the use of logarithmic functions in the field of *economics* are given below:

1. $P=40/\log_e x$.
   This is the *demand* function for a commodity where:
   x = the number of units of the given commodity which may be sold when
   P = the number of units of money in the price per unit of commodity.
2. $C=20\log_e x+3$.
   This is the *total-cost* function for a factory where:
   C = the total cost of producing the x units defined above.

Consider, finally, the use of the logarithmic function in a relationship of considerable interest to *social* scientists—the modern Weber-Fechner law in psychology. The law states that the strength of a response R to a situation varies directly as the logarithm of the strength of the stimulus S "inspiring" the response. In symbols, $R=k\log_e S$. Thus, in a given situation where $k=1$, the rising intensity of a response to an ever stronger stimulus would be faithfully reported by the familiar function $y=\log_e x$. The response could consist of the reaction to such "stimulating" events as:

1. Listening to a noise-maker.
2. Eating delicious foods.
3. Smelling unusual odors.
4. Touching or seeing someone we love. Perhaps, this explains why the modern Chinese "philosopher" may say: "Love is not *logic*al; rather, love is *log$_e$*-ical"!

Love and Population go Together Just Like the Exponential Function and the Logarithmic Function

It is highly instructive to *compare* the exponential and logarithmic functions. In order to facilitate the comparison of the two types of functions, attention may be restricted to the exponential function $y=e^x$ and the logarithmic function $y=\log_e x$.

Most helpful to observe is the *inverse* nature of the relationship between the exponential function $y=e^x$ and the logarithmic function $y=\log_e x$. Such a relationship recalls the inverse relationship which may exist between the square function $y=x^2$ and the square root function $x=y^{\frac{1}{2}}$ or between the linear functions $y=2x$ and $x=\frac{1}{2}y$ or between the transcendental functions $y=\cos x$ and $x=\text{arc}\cos y$.

Consider now Figure 1. Analysis of the figure reveals a *further*, striking relationship between the exponential and the logarithmic

functions. The line $y=x$ serves as a convenient "mirror" which permits the curves $y=e^x$ and $y=\log_e x$ to be "mirrored reflections" of each other. Hence, if the growth of population may be reflected by the function $y=e^x$ and if the growth of love may be reflected by the function $y=\log_e x$, then it may be mathematically seen from the figure that the growth of population may be reflected by the growth

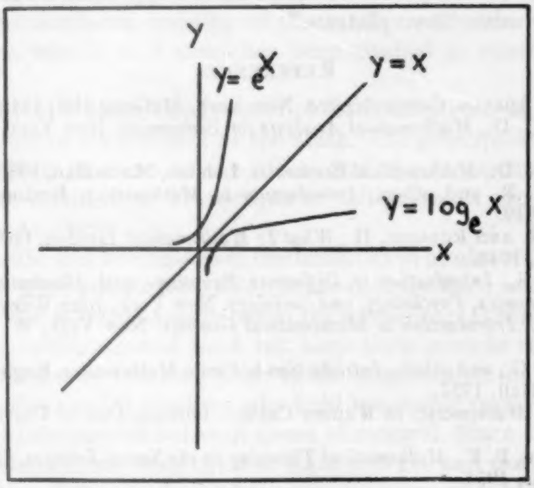

FIG. 1

of love. Thus, love and population may be viewed as "mirror images" and not "mirages"!

Now, even a cursory examination of the above figure may reveal that, in the case of each function graphed, as x increases, y increases. One might, therefore, hastily draw the conclusion that the "love curve" and the "population curve" are essentially *identical.* However, before jumping to such a conclusion, it may be well to consider for such curves the *rates* of growth.

Among its many wonderful contributions, the calculus provides a simple method of analysis of such rates of growth. Such an analysis reveals that the rates of growth for the logarithmic and exponential functions are very *different:*

1. In the case of $y=e^x$, the rate of change $(dy/dx)=y$ or e^x. Hence, in the case of our "population curve," the rate of growth of the population at a given instant equals the amount of the population. That is, as the population grows bigger, the *faster* will be the rate of growth of the population.
2. In the case of $y=\log_e x$, the rate of change $=1/x$ or $1/e^y$. Hence,

in the case of our "love curve," the rate of growth of the love response (a rate sometimes identified as the perceptibility of the sensation) equals the reciprocal of the strength of the stimulus to love. That is, as the love stimulus grows, the love response increases but at an ever *slower* rate. Thus, the mathematician may utilize the logarithmic function to describe the process of eventual "adaption" to love and the approach, at long last, to the elusive "love plateau."

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POLIO VIRUS WIDESPREAD THROUGHOUT U. S. NOW

The polio virus is now widespread in communities throughout the country, a U. S. Public Health official said.

Few areas of the country have, so far, escaped the effects of the virus. Paralysis has been reported from the north, south, east and west.

Health officials are concerned most about the preschool children. They are the number one candidates for this crippling summer disease.

Studies have shown that the Salk shots do not appear to break the chain of infection from the polio virus. This means that a person can be immunized against polio, yet carry the virus and infect others, some of whom will not be strong enough to build up a natural immunity. Paralysis can then occur.

The virus is usually accidentally swallowed by persons. Then it multiples in the digestive tract. From there it is passed on in secretions from the nose and mouth or excreted. The virus can even pass from one person to another during a kiss. If a person has the virus in his digestive tract, he will not necessarily have polio.

Academic Backgrounds of Kansas Mathematics Teachers

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One often hears the comment, "The high school teachers are not qualified to teach the courses they are teaching." Based on official records, the academic training of the mathematics teachers in the public high schools of Kansas has been studied in relation to this comment.[1]

The State of Kansas Department of Public Instruction at Topeka keeps records of all teachers of the state. The principals report the names of their teachers to the office, and the Departments issues appropriate certificates. Transcripts of college credit are on file for most of the teachers.

Of the 1,037 teachers teaching mathematics in grades 9 through 12, transcripts were found for 1,024. Some records were not complete which was due in most cases to one of these reasons: (1) some teachers hold a life certificate and need not keep their records current, (2) some teachers were teaching prior to the requirement for records in the state office, or (3) teachers who hold renewable certificates need not supply information between times of renewal. Since 48 per cent of the teachers came from the five state colleges and universities in Kansas, the registrars' offices at these institutions were visited to get all additional data available to make the results more complete.

The State Department of Public Instruction lists no specific subject preparation in mathematics, only the number of college hours needed for accreditation of the school in which the teacher teaches. The data were compiled and grouped in terms of hours in General Education Mathematics, Elementary (pre-analytic geometry courses), Intermediate (through calculus), Teaching of Mathematics, Theory of Equations, Abstract Algebra, College Geometry, Projective Geometry, Probability and Statistics, Advanced Analysis, and Applications.

Certain other background information was obtained. The results of the study are summarized below.

The records of 1,037 teachers of mathematics in grades 9 to 12, inclusive, during the academic year 1957–58 were studied. Of these records, 962 were complete. Fifty per cent of the teachers of mathematics teach in schools associated with communities 1,000 or less in population; 43.2 per cent teach in high schools with less than 100

[1] The entire study can be obtained from the Graduate Division, Kansas State Teachers College of Emporia, Kansas. John M. Burger, "Background and Academic Preparation of the Mathematics Teachers in the Public High Schools of Kansas, 1957–1958." *The Emporia State Research Studies*, Vol. 7, No. 3, March, 1959, p. 57.

students; 66.8 per cent teach in Class A high schools; and 64.4 per cent teach in a school system organized on the 8–4 plan.

There are 236 teachers of mathematics who teach only one class in mathematics, 410 who teach two or more classes but who also teach in some other field or are administrators, and 391 teachers who teach only mathematics.

Four hundred fifteen, or 40.0 per cent, of the teachers are 35 years old or less; 536, or 51.7 per cent, have 10 or less years of experience; and 685, or 66.1 per cent, have been in their present school systems 5 years or less.

Most of the teachers, 706, graduated from Kansas high schools, and 52.3 per cent graduated from high schools in towns of populations 2,000 or less. The annual salaries ranged from below $2,500 to over $6,000. The average salary was $4,386. Seventy-five per cent of the mathematics teachers received salaries between $3,501 and $5,000.

The most commonly taught subject was first year algebra, taught by 692 teachers. Plane geometry was taught by 461 teachers.

Three hundred twenty-six teachers of mathematics, or 31.4 per cent, teach only a single subject in mathematics. Most of these, 236 of the 326, teach only one section of this subject; the remainder of their teaching schedule being in other fields. Three hundred ninety-six teachers, or 38.2 per cent, teach two mathematics subjects; 214, or 20.6 per cent, teach 3 mathematics subjects; and the remainder, 101 or 9.7 per cent, teach 4 or more mathematics subjects.

Four hundred ninety-nine, or 48.1 per cent, of the teachers of mathematics in public high schools received their baccalaureate degrees from Kansas state colleges. An additional 286, or 27.2 per cent, received their degrees from other colleges in the state. Four hundred seventy-five, or 45.9 per cent, of the teachers have received their degrees within the past ten years. Four hundred ten teachers, or 39.5 per cent, have received master's degrees. Two hundred seventy-two of these degrees have been from Kansas institutions. Two hundred twenty-nine, or 55.1 per cent, of the degrees have been granted within the past 10 years.

The academic major for the bachelor's degree shows that 339, or 32.7 per cent, of the mathematics teachers majored in mathematics, while an additional 18.6 per cent have majored in science. For the master's degree, 234, or 69.4 per cent, had a major in education, and 12.5 per cent a major in mathematics. Four and seven-tenths per cent had a major in science.

The average *college* grade for mathematics teachers was B. The full-time mathematics teachers have somewhat better grades than those who teach part-time.

There are 21 teachers whose transcripts show no mathematics credit. However, 406 have over 24 college hours of mathematics, including one individual who has 76 college hours of mathematics.

The transcripts show that most teachers meet the current requirements for teaching in the class of school is which they are employed, 88.0 per cent of those in Class A, 84.5 per cent of those in Class B, and 83.2 per cent of those in Class C. There is no great difference in qualifications by age group, the average for the classes being 88.1 per cent qualified by current requirements. The variations by age group run from 83.3 per cent for the 61 years and over group to 92.8 per cent for the 21 to 25 years group.

Although the transcripts were read and the original data sheets were compiled in terms of specific courses, the preparation analysis was made in terms of groups of courses. Eight hundred thirty-eight teachers had completed college algebra and trigonometry, and 433 had completed calculus. Beyond these courses most teachers had no credit in other groups of subject matter.

Six hundred seventy-nine had no credit in courses which are specifically designed as academic background for teachers (Teaching of Secondary School Mathematics, History of Mathematics, etc.), excluding practice teaching in mathematics; 801 had no geometry in college; 839 had no advanced algebra course; while 863 had no courses in probability and statistics.

According to the available transcripts there are 21 teachers with no college credit in mathematics. They teach 723 mathematics students.

In addition, there are 302 more teachers with 1 to 14 hours of college mathematics, teaching 13,219 mathematics students, for a total of 13,942 students taught by teachers with less than 15 hours of college mathematics.

More accuracy in a survey of this type requires a complete current file of academic records for all teachers. As a means of maintaining these records up to date, the renewable certificate will be somewhat effective.

Academic preparation of teachers in itself is only one of the qualifications of an individual to do good teaching, of course, because other items must be included. Certainly attitude toward the teaching profession, attitude toward students, teaching load, and demands of extra curricular duties are all factors in determining the effectiveness of the teacher in his classroom. Nevertheless, academic preparation is important, and the data contained in this study attempt to show some aspects of this subject as it currently exists in the public high schools of Kansas.

Conclusions

Most mathematics teachers (62 per cent) teach in some other field as well as mathematics; this includes 23 per cent who teach only a single class in mathematics.

One-half of the mathematics teachers have ten years or less of experience.

The state requirements for teaching mathematics are being raised from 15 college hours of mathematics (or 9 hours with 3 years of high school) for class A, 12 hours (or 6 with 3 years of high school) for class B, and 8 hours (or 6 with 1 year of high school) for class C to 18 hours for a standard with a minimum of 15 hours. No high school deductions will be allowed. This will greatly lower the per cent of teachers meeting the current standards unless they return to college for further training as only 68.4 per cent of the teachers currently have fifteen or more hours of college mathematics.

The minimum college hour requirements for teaching mathematics have been lower than for other fields nationally. There has been no explanation of this situation in any reports the author has read.

In the opinion of the author no reasonable justification can be be found for assigning classes to unqualified teachers especially when the requirements have been so low.

Part of the shortage of qualified teachers is due to misassignment of teaching load. Although not shown in the above summary, the report shows that many qualified mathematics teachers have moved into administrative positions, thus being lost to classroom teaching. One-fifth of the high school students in mathematics are taught by teachers with less than 15 semester hours of college mathematics.

The state has issued certificates without field of qualification endorsement, and this makes checking of classification of schools in terms of teacher qualifications very difficult.

Recommendations

Some recommendations for improving the academic background of mathematics teachers in Kansas, are:

1. Field endorsement on teaching certificate.
2. Requirement to meet new standards as they are changed.
3. Co-operation among small schools to make qualified mathematics teachers available by sharing a teacher, i.e. part time at each school.
4. Consolidation of schools to increase student body, so that full time specifically trained personnel may be hired and used effectively.

Teaching Rate and Ratio in the Middle Grades*

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After reading the title of this article you may wonder about the propriety of teaching ratio in the middle grades because the topic of ratio is typically deferred until the upper grades. The recommendation that we teach ratio in the middle grades is based upon the conviction that ratio is such an important concept that we need to develop the concept carefully and thoroughly. If we use learning experiences similar to those described later in this paper, we will find that the concept of ratio can be learned by pupils in grades four, five, or six without any difficulty. It seems likely that many pupils in grades seven or eight have difficulty at the present time in learning about ratio because of the *way* we try to teach it rather than because of any inherent difficulty of the concept itself. Another reason for including the topic of ratio in the middle grades is that ratio is very useful in solving most problems that lead to multiplication or division. Ratio and proportion constitute a general attack for all such problems. Finally, since per cent is a special way of expressing a ratio, a good understanding of ratio makes the learning of per cent much easier. Research by McMahon[1] has demonstrated that for seventh graders a ratio approach to per cent is significantly better than the traditional approach. It seems very reasonable to infer that instruction in ratio prior to the seventh grade would have made even greater differences in favor of the ratio approach.

The Concept of Ratio

In the past ratio has been presented as a fraction. Rather than assume that ratio is a fraction, let us examine some concrete situations and see how the concept of ratio emerges.

Example A: A recipe for the batter of burnt sugar cake calls for $1\frac{1}{2}$ cups of sugar and $2\frac{1}{2}$ cups of cake flour among other things. If the recipe were doubled, one would need 3 cups of sugar and 5 cups of cake flour. The balance between sugar and flour would be preserved even though the amounts were doubled.

Example B: A garden supply store sells a grass seed mixture packaged into $1\frac{1}{2}$ pound bags. The price for a bag is \$2.50. In this case $1\frac{1}{2}$ pounds are associated with $2\frac{1}{2}$ dollars. If one would buy two bags of seed, he would get 3 pounds for 5 dollars. Notice that the

* Based on the author's speech at the 59th Annual CASMT Convention, Chicago, Illinois, November 26–28, 1959.

[1] McMahon, Della, *An Experimental Comparison of Two Methods of Teaching Per Cent.* Ed.D. Dissertation, University of Missouri, 149 pp., 1959.

numbers involved in this situation ($1\frac{1}{2}$, $2\frac{1}{2}$, 3, and 5) are the same as in the cake recipe situation.

Example C: Another situation involving the same numbers might be that of a person traveling along in an automobile. He travels at a steady rate; namely, each $1\frac{1}{2}$ miles takes $2\frac{1}{2}$ minutes. To travel 3 miles would require 5 minutes.

In each of these three situations there is a correspondence of so-much of something to so-much of something. The "something" may be volume (or capacity), weight, money, distance (or length), or time as in the three illustrative situations. The "something" may also be area, temperature, angle, or any group of separate objects. The correspondence or association of the two quantities in each situation must be maintained. If one quantity is doubled, the other quantity must also be doubled to maintain the correspondence. If one quantity is halved, so must the other quantity be halved. It will help communication if we agree upon a term for the correspondence between two quantities—let us call it a *rate* since some of the possible correspondences are already called that.

If we examine other concrete situations, we may discover many rates. Rates involving time and money are very common. People who earn a wage or a salary express the wage or the salary as so-much money per so-much time. In the United States, the monetary unit used is the dollar and the units of measure for time may be hour, day, week, month, academic year, or calendar year. Some examples are: \$1.50 per hour ($1\frac{1}{2}$ dollars per 1 hour), \$96 per week (96 dollars per 1 week), and \$5,260 per year (5,260 dollars per 1 year). Rates involving weight and money are encountered in buying or selling many products. Some examples are: 2 pounds of hamburger for 89 cents, 1 ton of coal for $9\frac{1}{4}$ dollars, 5 pounds of sugar for 59 cents, and 10 pounds of potatoes for 98 cents. Rates involving distance and time are traditionally called rates of speed. Some examples are: 35 miles per 1 hour, 24 feet per 1 second, and 450 miles per 1 day. Atmospheric pressure is expressed in the rate of weight per area; density is expressed in the rate of weight per volume; and land is sold at the rate of money per area. The rates mentioned here are but a few of the possibilities. Perhaps you are ready to agree that rate situations are very common.

The examples which have been given show something else, too. The size of a quantity (involved in a rate) is expressed by the number of a certain unit of measure. That is, the "so-much of something" is described by "so-many of a unit of measure." The unit of measure may be a standard unit such as pound, hour, dollar, or acre, or it may be a non-standard unit such as a bunch, a can, or any individual object such as a grapefruit or a handkerchief.

Any expression of a rate involves two numbers and two units of measure. The examples given indicate that the two units of measure frequently are different. An interesting pattern of the numbers develops as the rate is expressed by various names. There is, however, a condition that must be met in order for the pattern to develop. The units of measure must not be changed as we use different names for the rate. This means that in Example C, we must not change the units of mile and minute. We must not change the unit of mile to the unit of feet for then the rate would be expressed as 7,920 feet per $2\frac{1}{2}$ minutes, which would involve the numbers 7,920 and $2\frac{1}{2}$ instead of the numbers $1\frac{1}{2}$ and $2\frac{1}{2}$. Nevertheless, there are many names that could be used to express the rate confining the units of measure to mile and minute. For example, $1\frac{1}{2}$ miles per $2\frac{1}{2}$ minutes, 3 miles per 5 minutes, $4\frac{1}{2}$ miles per $7\frac{1}{2}$ minutes, 6 miles per 10 minutes, 9 miles per 15 minutes, etc., are all different names for the same rate. Note the pattern of the numbers involved in these expressions: $(1\frac{1}{2}, 2\frac{1}{2})$, (3, 5), $(4\frac{1}{2}, 7\frac{1}{2})$, (6, 10), and (9, 15), etc. Note also that exactly the same pattern occurs with different expressions of the rate of sugar to cake flour in Example A and with different expressions of the rate of grass seed to price in Example B.

When we pull just the number pairs from a rate situation we ignore the units of measure. The result is an abstraction which can be recognized as a *ratio*. The ratio used in Examples A, B, and C can be expressed in many different ways, but perhaps the simplest name is 3 per 5, or 3 to 5. Some other names for the same ratio are 6 per 10, $4\frac{1}{2}$ per $7\frac{1}{2}$, 30 per 50, and 60 per 100.

It is interesting and instructive to notice that the abstraction of rate situations to get a ratio is similar to the abstraction of groups of objects to get a number. That is, we engage in a similar kind of abstraction when we consider a group of four chairs, a group of four people, a group of four quarts of milk, and a group of four apples and call the common property or quality of these groups the number four.

In the past pupils were admonished to compare quantities measured with the same unit of measure. Example A would satisfy this admonition because the unit of cup is used to measure the sugar as well as to measure the flour. However, Examples B and C would not satisfy the requirement that the units used in the comparison be the same. These examples and the development of ratio out of rate situations show that the requirement (to use the same unit of measure when comparing) is entirely unnecessary. In fact, the full power of ratio is not achieved with such a restriction. Since a ratio involves only pairs of numbers, the rate situation which uses these pairs of numbers can involve any pair of units of measure.

Materials and Activities for Teaching Rate and Ratio

Most pupils have had some experience in rate situations even before attending school. Perhaps his family has cookies for dessert and each member of the family is given two cookies. Here the rate involves separate objects rather than continuous-type quantities. The correspondence is between groups of cookies and groups of persons. The rate could be expressed as 2 cookies per 1 person, 4 cookies per 2 persons, 6 cookies per 3 persons, etc.

The first activities in school that we use to develop the concept of rate should involve groups of separate objects not only because this is similar to a pupil's past experience but also because there is not the complicating extra matter of measuring a quantity. We can build up situations in the classroom similar to this one: A certain kind of candy is sold 2 pieces for 5 cents. We may use actual candy and one-cent coins for this situation or we may use felt cut-outs on a flannel board or some other representative materials—a diagram sketched on the chalk-board, or paste sticks and checkers, or the like. With these materials it is advisable to show first the correspondence of 2 pieces of candy to 5 cents. Then show another group of 2 pieces of candy matched to another group of 5 cents. Following this the groups of candy can be combined and likewise the groups of coins. The result is a matching of 4 pieces of candy to 10 cents. In a similar way three sets of 2 pieces of candy matched with 5 cents can be combined to get a matching of 6 pieces of candy with 15 cents. At this stage we may symbolize on paper or the chalk-board the rate by writing

$$\frac{2 \text{ candies}}{5 \text{ cents}} \text{ and } \frac{4 \text{ candies}}{10 \text{ cents}} \text{ and } \frac{6 \text{ candies}}{15 \text{ cents}}.$$

After some other situations (12 eggs per 1 carton, 4 pieces of glass per 1 window, 6 crayons per 1 pupil, 4 legs per 1 dog, etc.) are developed in the classroom in a fashion similar to the one just described, we may then attach the label of *rate* to any of these matchings. Notice that the activities explained so far could be carried out before the concept of fractions has been developed. That does not mean that these activities should come before activities for developing fractions, however. Experimentation is needed to help determine the best sequence.

After some of these activities for building up the notion of rate have been carried out, we may then introduce the equality sign. Even though the equals sign may have been used in the past with the pupils, it is helpful to extend the meaning of the equals sign in this way: whenever we have two names or marks for the same thing we may put the equals sign between the two names. The statement

"James = Jim" is then interpreted to mean that "James" and "Jim" are two names for the same boy. Likewise, the statement "4+7=11" means that "4+7" is a name or mark for a number and that "11" is another name or mark for the same number. With this understanding about the meaning of the equality sign, we may help the pupils to see that "2 candies for 5 cents" and "4 candies for 10 cents" are two names for the same rate. We can then write

$$\frac{\text{2 candies}}{\text{5 cents}} = \frac{\text{4 candies}}{\text{10 cents}}.$$

Notice that we are not saying that two-fifths equals four-tenths (even though that would be a true statement). The inclusion of the names of the units, "candies" and "cents," helps to distinguish between these two different statements.

Following these activities for developing the concept of rate, we may organize some activities for abstracting and thus developing the concept of ratio. Such activities might be planned incorporating rate situations like Examples A, B, and C explained earlier. The crucial aspect of the rate situations used is that the situations must involve the same number pairs in each situation. The pairs of units of measure used in the situations ought to be different from situation to situation, however. The pupils can be asked to look for similarities in the rate situations. When they discover the same pattern for the number pairs, they will have achieved the beginning part of the concept of ratio. The term ***ratio*** is appropriately applied to the concept at this time. A ratio may be symbolized in the forms, 3 to 5, 3 per 5, 3:5, and $\frac{3}{5}$. The latter symbol is never read as three-fifths when referring to a ratio, however. It should be read "3 to 5" or "3 per 5" or "3 out of 5." A statement that two names are names for the same ratio (such as $\frac{3}{5}=\frac{6}{10}$) is also appropriately called a ***proportion*** at this stage.

The Use of Rate and Ratio in Solving Problems

As it was stated earlier, one reason for including the topics of rate, ratio, and proportion in the middle grades is that most problems that lead to multiplication or division can be solved by using ratio and proportion. Ratio and proportion can be used in connection with solving these problems because the problem-situations have a rate structure. That is, the problems actually involve rates.

First, let us examine a fairly typical multiplication problem: "Miss Brown wishes to give each pupil in her class 3 sheets of construction paper. There are 24 pupils in her class. How many sheets of paper will she need for this?" This problem-situation involves the known rate of

3 sheets per 1 pupil, and what we are looking for is a part of another name for this rate. That is, using n as a symbol for the unknown number of sheets needed for the whole class, we can write the statement:

$$\frac{3 \text{ sheets}}{1 \text{ pupil}} = \frac{n \text{ sheets}}{24 \text{ pupils}} .$$

Such a statement might be called a *quasi-proportion* since it is similar to a proportion. In the early stages, the quasi-proportion can be used effectively instead of the more abstract proportion. The problem is not yet solved, but the essential structure of the problem has been determined and then described by the quasi-proportion. The quasi-proportion can then be used by a pupil to help decide which numbers should be multiplied or which numbers should be divided.

At this point in the paper, it is necessary to digress to some extent and consider how pupils can learn to solve a quasi-poportion or a proportion. With reference to Example A, a pattern was detected in the number pairs, ($1\frac{1}{2}$, $2\frac{1}{2}$), (3, 5), ($4\frac{1}{2}$, $7\frac{1}{2}$), (6, 10), and (9, 15), etc. The pattern could be described in terms of how any two of the number pairs are related to each other. By using leading questions, we may help the pupils to discover that if both numbers in a number pair are multiplied by a number, the resulting number pair expresses the same ratio. That is, when both 3 and 5 in the pair (3, 5) are multiplied by 2, we get the pair (6, 10). Likewise if both numbers in a number pair are divided by a number, the resulting number pair is another name for the same ratio. Thus when both 6 and 10 in (6, 10) are each divided by 4, the resulting pair ($1\frac{1}{2}$, $2\frac{1}{2}$) represents the same ratio. The generalization that "when both numbers in a ratio are multiplied or divided by a number different from zero the result is another name for the same ratio" is all that is needed to solve proportions. A similar generalization can be made for rates prior to the one for ratios if desired.

Returning to the quasi-proportion,

$$\frac{3 \text{ sheets}}{1 \text{ pupil}} = \frac{n \text{ sheets}}{24 \text{ pupils}} ,$$

we can find the unknown number in the statement by thinking along these lines: The number 1 would have to be multiplied by 24 to get the number 24 in the expression for the rate at the right, so we should multiply 3 by 24 to get the other number in the rate "n sheets per 24 pupils." Thus the problem is solved when 3 is multiplied by 24.

Second, let us examine a typical measurement-type problem situation leading to division: "If pencils cost 5 cents each, how many

pencils could Jack buy with 30 cents?" Again, there is a known rate, 5 cents per 1 pencil. the structure of the problem can be described by the statement:

$$\frac{5 \text{ cents}}{1 \text{ pencil}} = \frac{30 \text{ cents}}{n \text{ pencils}}.$$

The thinking that would enable a pupil to find the unknown number might be: 5 multiplied by something gives 30. I can divide 30 by 5 to find this "something." The result of dividing 30 by 5 is 6. Since 5 multiplied by 6 gives 30, the other number in the rate, 1, should also be multiplied by 6. 1 multiplied by 6 is 6, so the unknown number is 6 and the problem is solved.

Third, let us consider a typical partition-type problem situation leading to division: "Six boys decide to share equally the candy in a box. There are 24 pieces of candy in the box. How many pieces of candy should each boy get?" Again, there is a known rate, 24 candies for 6 boys. The problem situation can be described by the statement:

$$\frac{24 \text{ candies}}{6 \text{ boys}} = \frac{n \text{ candies}}{1 \text{ boy}}.$$

This statement can help a pupil's thinking when he reasons that 6 would have to be divided by 6 to give 1, so the other rate number, 24, should also be divided by 6 to give the solution for the problem.

Fourth, let us examine a sort of compound problem situation: "If soup is selling at 3 cans for 55 cents, how much would 12 cans cost?" The statement,

$$\frac{3 \text{ cans}}{55 \text{ cents}} = \frac{12 \text{ cans}}{n \text{ cents}},$$

describes this problem. A pupil could then think: 3 would have to be multiplied by 4 to get 12 (I could divide 12 by 3 to find the number 4), so 55 should also be multiplied by 4.

The preceding problems were solved with the aid of quasi-proportions. My suggestion is to use quasi-proportions for the initial activities in solving such problems and then later use proportions instead. Also, when the pupils seem to understand how to find the unknown number in a porportion, it would be advisable to help pupils discover that the two "cross-products" must always be the same for any particular proportion. Using cross-products is more efficient than the approach described earlier.

Not all of the possible uses of rates, ratios, and proportions have been described in this paper. Perhaps enough uses have been presented to show the importance of laying a good foundation for these

concepts in the middle grades. An added benefit is that these concepts help pupils tie together many of the things that he learns about arithmetic in the middle grades. I hope that you will be eager to learn more about this exciting new development in the arithmetic curriculum.

NATIONAL SCIENCE FOUNDATION ANNOUNCES 379 SUMMER INSTITUTES FOR HIGH SCHOOL AND COLLEGE TEACHERS OF SCIENCE, MATHEMATICS, AND ENGINEERING

Financial aid will be available in 1960 for about 18,000 high school and college teachers of science, mathematics, and engineering to participate in Summer Institutes sponsored by the National Science Foundation. Three hundred and seventy-nine Institutes will be supported by the Foundation next summer, in 265 educational institutions.

Awards of grants totaling more than $21,000,000 for the Summer Institutes were announced today by Alan T. Waterman, Director of the National Science Foundation. Institutes will be held in all 50 states, the District of Columbia and Puerto Rico. Three hundred and sixteen of the institutes will be open to high school teachers only, 37 will be for college teachers only, and 24 will be for both high school and college teachers, and two will be for technical institute personnel. Roughly 16,000 high school teachers and 2,000 college teachers will be enabled to participate through stipends provided by the National Science Foundation.

The success of previous Summer Institutes in meeting the needs of teachers is a major factor contributing to the growth of the program. The first two Summer Institutes supported by the National Science Foundation were held in 1953. The number has grown each summer, reaching 125 in 1958, 350 in 1959 and 379 next summer.

Seventeen institutes offering courses in radiation biology for high school teachers, and five similar institutes for teachers in small colleges, are being jointly sponsored by the Foundation and the Atomic Energy Commission, as are three institutes in isotope technology for college teachers.

The number of teachers who will receive financial support in each of the 379 institutes will average nearly 50, and will vary from 15 to more than 100. Tuition and fees will be paid for these teachers. They will also receive stipends of not more than $75 per week for the duration of the institute, plus allowances for travel and dependents. The institutes will vary in length from four to twelve weeks.

Application blanks are obtained only from host institutions and NOT from the National Science Foundation. The complete application blanks (including in every case a card for the National Science Foundation) must be postmarked by *February 15, 1960*, to guarantee consideration. Institutes will make initial stipend offers on or before *March 15, 1960. In every case, recipients will have until April 1, 1960, to accept or decline.*

NATIONAL SCIENCE FOUNDATION

SUMMER INSTITUTES

1960

For High-School Teachers Only, except as follows:
* For College Teachers Only.
† For High-School and College Teachers.
♯ For Teachers in Technical Institutes and in Technical Curricula in Junior Colleges.

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[1] One Institute for high school teachers; one for high school and college teachers.

[2] One Institute for high school teachers; one Institute for college teachers.

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[1] One Institute for high school teachers; one Institute for college teachers.

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PROBLEM DEPARTMENT

Conducted by Margaret F. Willerding

San Diego State College, San Diego, Calif.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the probelms should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the Department desires to serve her readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, San Diego State College, San Diego, Calif.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Solutions should be in typed form, double spaced.

2. Drawings in India ink should be on a separate page from the solution.

3. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

4. In general when several solutions are correct, the one submitted in the best form will be used.

LATE SOLUTIONS

2676. *John Stafford, London, England.*

2684. *J. Barry Love, St. Davids, Pa.*

2685. *W. R. Talbot, Jefferson City, Mo.*

2689. *Proposed by Cecil B. Read, University of Wichita, Wichita, Kans.*

Prove that if the ratio $(z-i)/(z-1)$ is purely imaginary the point z lies on the circle whose center is at the point $(1+i)/2$ and which has a radius $\sqrt{2}/2$.

Solution by W. R. Talbot, Jefferson City, Mo.

Let the ratio be ik and let $z=x+iy$. Then $z-i=ikz-ik$ or

$$(x+ky)+i(y-1-kx+k)=0.$$

Then $x+ky=0$ and $y-1-kx+k=0$. Eliminating k between these two equations gives the circle

$$y^2-y+x^2-x=0 \quad \text{or} \quad (x-1/2)^2+(y-1/2)^2=1/2.$$

The radius is $\sqrt{2}/2$ and in complex form the center is $(1+i)/2$.

Solutions were also submitted by William G. Koellner, Hillside, N. J.; Wayne Shepherdson, Modesto, Calif.; and the proposer.

2690. *Proposed by Leo Moser, University of Alberta.*

In how many ways can a king go from the left corner of a chessboard to the right upper corner, if the permissible moves are single steps horizontally to the right, vertically up, and diagonally up and to the right?

Solution by the Proposer

If a path involves r diagonal steps, then it will involve $7-r$ horizontal moves

and $7-r$ vertical moves. We consider first the number of paths involving exactly r diagonal moves. By a well known combinatorial principle, the number of ways of arranging n things, which consist of a of one type, b of a second and c of a third is $n!/a!b!c!$. Hence the number of paths with r diagonal steps will be

$$\frac{(r+(7-r)+(7-r))!}{r!(7-r)!(7-r)!}=\frac{(14-r)!}{r!(7-r)!^2}.$$

Hence the total number of paths will be

$$\sum_{r=0}^{7}\frac{(14-r)!}{r!(7-r)!^2}=48639.$$

Solutions were also offered by Julian Braun, Seattle, Wash;. and W. R. Talbot, Jefferson City, Mo.

2691. *No solution has been offered.*

2692. *Proposed by J. B. Flansburg, Houston, Texas.*

The fifty-two cards of a "well-shuffled" deck are laid out in a row. Directly below these are placed the cards of another deck, similarly prepared. What is the probability that there will be at least one matching of two like cards?

Solution by Julian H. Braun, Seattle, Wash.

In the French gambling game Treize, a bridge deck is shuffled and the cards are turned up one at a time. As they are turned the dealer counts from one to thirteen, four times. If the denomination of a card coincides with the number called, it is called a "hit." In a modified version the count may be made up to 52. This becomes equivalent to the present problem when it is realized that the second deck could just as well be a new unshuffled deck. If Treize is played with a deck of n cards numbered from 1 to n, there are exactly $n¡$ (*n subfactorial*)* ways of getting no hits. Since there are $n!$ possible arrangements the probability of no hits is exactly $n¡/n!$ This ratio rapidly approaches its limiting value, e^{-1}, with increasing n. The probability of at least one hit with 52 cards is

$$1-\frac{52¡}{52!}\doteq 1-e^{-1}\doteq .6321,$$

correct to four decimal places.

Solutions were also offered by William G. Koellner, Hillside, N. J.; John Pilaar, Howe, Ind.; W. R. Talbot, Jefferson City, Mo.; and Lowell Van Tassel, San Diego, Calif.

2693. *Proposed by Lloyd A. Walker, San Mateo, Calif.*

Find the "volume" common to n mutually perpendicular intersecting cylinders in n-dimensions each of radius "a" and having their axes concurrent.

Solution by the Proposer

This is an n-dimensional space problem which can be solved by using analogous reasoning to that of the corresponding 3-dimensional case.

Getting the answer is simply a matter of using Cavalieri's Theorem and enumerating correctly the various parts of the n-dimensional solid. However, it will become obvious that the sigma notation answer given below applies only to a space of an odd number of dimensions. The problem is an excellent exercise in the use of symbolism of the sigma and the binomial coefficients.

The solution is:

$$\text{"}V_T\text{"}=\frac{(2a)^n}{(n-1)^{n/2}}+2^n na^n\sum_{r=0}^{r=(n-1)/2}(-)\frac{r\binom{(n-1)/2}{r}\left[(n-1)^{(2r+1)/2}-1\right]}{(2r+1)(n-1)^{(2r+1)/2}}$$

* See Webster's Unabridged Dictionary for a definition of *subfactorial*.

where n is the number of dimensions and a is the radius of each of the concurrent cylinders. The formula is easily checked for $n=3$ to give the correct value for three mutually perpendicular cylinders:

$$V_T=8a^3(2-\sqrt{2})$$

For $n=5$ we have also:

$$\text{“}V_T\text{”}=\frac{56a^5}{3}$$

2694. *Proposed by Cecil B. Read, University of Wichita, Wichita, Kans.*

Determine the real values of x for which

$$\frac{1}{x}+\frac{1}{x+1}$$

exceeds $\frac{1}{2}$.

Solution by the proposer

$$\frac{1}{x}+\frac{1}{x+1}>\frac{1}{2}$$

when and only when

$$\frac{2x+1}{x(x+1)}>\frac{1}{2}\cdot$$

Clearing of fractions, we have:

If $x(x+1)>0$

$$4x+2>x^2+x$$

or $3x+2>x^2$

or $x^2-3x-2<0$

or $(x-3/2)^2<17/4$

Hence x must satisfy the conditions: outside the range

$$-1\leq x\leq 0 \quad \text{and}$$

inside the range

$$\tfrac{1}{2}(3-\sqrt{17})<x<\tfrac{1}{2}(3+\sqrt{17})$$

but if $x(x+1)<0$

$$4x+2<x^2+x$$

or $3x+2<x^2$

or $x^2-3x-2>0$

or $(x-3/2)^2>17/4$

Hence x must satisfy the conditions: inside the range

$$-1<x<0 \quad \text{and}$$

outside the range

$$\tfrac{1}{2}(3-\sqrt{17})\leq x\leq\tfrac{1}{2}(3+\sqrt{17})$$

Combining the results above, we find two ranges of acceptable values for x:

$$0<x<\tfrac{1}{2}(3+\sqrt{17}) \quad \text{and} \quad -1<x<\tfrac{1}{2}(3-\sqrt{17})$$

Solutions were also offered by Julian Braun, Seattle, Wash.; Robert Guderjohn, Pocatello, Idaho; William G. Koellner, Hillside, N. J.; John Pilaar, Howe, Ind.; W. R. Talbot, Jefferson City, Mo.; Floyd D. Wilder, Bethany, Oka.; and Dale Woods, Kirksville, Mo.

STUDENT HONOR ROLL

The Editor will be very happy to make special mention of classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each student contributor will receive a copy of the magazine in which his name appears.

PROBLEMS FOR SOLUTIONS

2713. *Proposed by Walter R. Talbot, Jefferson City, Mo.*

Verify the inequality

$$\frac{1}{4}<\frac{\sqrt{5+1}}{5\sqrt{10-2\sqrt{5}}}<\frac{\sqrt{3}}{6}$$

2714. *Poroposed by Cecil B. Read, Wichita, Kans.*

The numbers 1, 2, 3, $\cdots$, $2n$ are arranged consecutively around a circle at equal intervals. Through one of these numbers, h, a diameter is drawn. For what values of n and h will the sum of the numbers on one side of the diameter be equal to the sum of those on the other?

2715. *Proposed by Leo Moser, University of Alberta.*

Five lines of general position in a plane meet in 10 points. Show that it is not possible to enter the numbers 1, 2, $\cdots$, 10 on these points in such a way that the sum of the numbers along every line is the same.

2716. *Proposed by Brother Felix John, Philadelphia, Pa.*

The following equations have a common root. Determine a, b, and c, if possible.

$$ax^2+3x+c=0$$
$$3x^2+bx+c=0$$
$$2ax^2+(b+2)x-2=0.$$

2717. *Taken from* Ladies' Diary, *1810.*

What is the smallest square number which, when squared, results in the largest possible succession of equal digits?

2718. A mathematician gives a small stag party. He invites his father's brother-in-law, his brother's father-in-law, his father-in-law's brother, and his brother-in-law's father. Find the number of guests.

Adapted from Charles L. Dodgson

BACTERIA USED TO EXTRACT OIL FROM SANDS AND SHALE

Bacteria are used to extract oil from oil-bearing sands and shales in a new recovery process.

Recovery of oil from such inorganic solids has always been a problem because of the close association of the oil with the sand or shale and the necessity of handling large quantities of the solids. Previous processes, including destructive distillation, solvent extraction, and hydrogenation at high temperatures and pressures in the presence of a catalyst, have not satisfactorily overcome these problems and give relatively low yields.

The "improved" process uses a combination of hydraulic mining and bacteria in the presence of oxygen. The mechanism by which bacteria displace oil from solids is not clearly understood, but certain aerobic bacteria have been found to very adept at it. This is how his process works:

The oil sand or shale is mined by the hydraulic action of water so that the material is slurried in the water. Then it is washed to an aerated accumulating zone, such as a pond. Bacteria placed in the pond grow in the presence of oxygen and act to release oil from the sand or shale. When a sufficient amount of oil has been released, the sand becomes water wet and sinks to the bottom. The oil left floating on the surface is recovered, usually by a skimming operation.

The water from the pond may be recycled for additional mining and slurring because it contains active bacteria that aid in loosening the oil from the soil during the hydraulic mining operation.

Books and Teaching Aids Received

THE SUN, THE MOON, AND THE STARS, by Mae and Ira Freeman, Cloth, Pages 83. 22.5×16.5 cm. 1959. Random House, 457 Madison Ave., New York 22, N. Y. Price $1.95.

MODERN ELECTRONIC COMPONENTS, by G. W. A. Dummer. Cloth. Pages viii +472. 21.5×14 cm. 1959. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $15.00.

ROCKS ALL AROUND US, by Anne Terry White. Cloth. Pages 82. 22.5×16.5 cm. 1959. Random House, 457 Madison Ave., New York 22, N. Y. Price $1.95.

SIMPLE MACHINES AND HOW THEY WORK, by Elizabeth N. Sharp. Cloth. Pages 83. 22.5×16.5 cm. 1959. Random House, 457 Madison Ave., New York 22, N. Y. Price $1.95.

ROCKETS INTO SPACE, by Alexander L. Crosby and Nancy Larrick. Cloth. Pages 82. 22.5×16.5 cm. 1959. Random House, 457 Madison Ave., New York 22, N. Y. Price $1.95.

THE MODERN SLIDE RULE, by Stefan Rudolf. Paper. Pages 69. 27.5×21.5 cm. 1959. The William-Frederick Press, 391 East 149th Street, New York 55, N. Y. Price $5.00.

LOGIC IN ELEMENTARY MATHEMATICS, by Robert M. Exner and Myron F. Rosskopf. Cloth. Pages xi+274. 22.5×15 cm. 1959. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N. Y. Price $6.75.

IN THE DAYS OF THE DINOSAURS, by Roy Chapman Andrews. Cloth. Pages 80. 22.5×16.5 cm. 1959. Random House, 547 Madison Avenue, New York 22, N. Y. Price $1.95.

NEW DIMENSIONS OF FLIGHT, by Lewis Zarem. Cloth. Pages 256. 24×16.5 cm. 1959. E. P. Dutton and Company, 300 Park Ave. South, New York 10, N. Y. Price $4.50.

THE CHEMICAL ELEMENTS, by Helen Miles Davis. Paper. Pages 198. 18×10.5 cm. 1952, 1959. Science Service, Inc., 1719 N. St. N.W., Washington 6, D. C., Ballantine Books, Inc., 101 Fifth Ave., New York 3, N. Y. Price $.50.

SHORTWAVE PROPAGATION, by Stanley Leinwoll. Paper. Pages 160. 21.5×14 cm. 1959. John F. Rider Publisher, Inc., 116 West 14th Street, New York 11, N. Y. Price $3.90.

R-F AMPLIFIERS, Edited by Dr. A. Schure, Ph.D. Paper. Pages 104. 21.5×14 cm. 1959. John F. Rider Publisher, Inc., 116 West 14th Street, New York 11, N. Y. Price $2.40.

THE SCIENTIFIC AMERICAN BOOK OF MATHEMATICAL PUZZLES AND DIVERSIONS, by Martin Gardner. Cloth. Pages xi+178. 21×14 cm. 1959. Simon and Schuster, Inc., Rockefeller Center, 630 Fifth Avenue, New York 20, N. Y. Price $3.50.

SYMPOSIUM ON BASIC RESEARCH, by Dael Wolfle. Cloth. Pages xvii+308. 23 ×15.5 cm. 1959. The American Association for the Advancement of Science, 1515 Massachusetts Ave., Washington, D. C.

MATHEMATICAL ANALYSIS, by Edwin M. Hemmerling. Cloth. Pages xi+332. 21×14 cm. 1959. McGraw-Hill Book Co., Inc., 330 West 42 Street, New York, 36, N. Y. Price $5.75.

EDUCATORS GUIDE TO FREE TAPES, SCRIPTS, AND TRANSCRIPTIONS, by Walter A. Wittich, Ph.D. and Gertie Hanson Halsted, M.A. Paper. Pages xiv+225.

27.5×21.5 cm. 1960. Educators Progress Service, Randolph Wisconsin. Price $5.75.

COLLEGE REGISTRAR AS A CAREER, No. 103 IN VOCATIONAL AND PROFESSIONAL MONOGRAPHS, by Sylvia Dean Herbert. Paper. Pages 20. 23×15.5 cm. 1959. Bellman Publishing Company, Post Office Box 172, Cambridge 38, Massachusetts.

CHEMISTRY AS A PROFESSION, No. 104 IN VOCATIONAL AND PROFESSIONAL MONOGRAPHS, by Dr. J. L. Riebsomer, Paper. Pages 20. 23×15.5 cm. 1959. Bellman Publishing Company, Post Office Box 172. Cambridge 38, Massachusetts.

CLIFF DWELLERS OF WALNUT CANYON, by Carroll Lane Fenton and Alice Epstein. Cloth. Pages 63. 20.5×17 cm. 1960. The John Day Company, Inc., 210 Madison Avenue, New York, N. Y. Price $2.75.

RETAILING AS A CAREER, No. 22 IN VOCATIONAL AND PROFESSIONAL MONOGRAPHS, by J. Gordon Dakins. Paper. Pages 52. 23×15.5 cm. 1959. Bellman Publishing Company, Post Office Box 172, Cambridge 38, Massachusetts.

SECRETARIAL SCIENCE, No. 50 IN VOCATIONAL AND PROFESSIONAL MONOGRAPHS, by Mildred J. Langston. Paper. Pages 29. 23×15.5 cm. 1959. Bellman Publishing Company, Post Office Box 172, Cambridge 38, Massachusetts.

PHARMACY, No. 51 IN VOCATIONAL AND PROFESSIONAL MONOGRAPHS, by Earl P. Guth. Paper. Pages 24. 23×15.5 cm. 1959. Bellman Publishing Company, Post Office Box 172, Cambridge 38, Massachusetts.

THE CANNING INDUSTRY ,No. 99 IN VOCATIONAL AND PROFESSIONAL MONOGRAPHS, by Nelson H. Budd. Paper. Pages 36. 23×15.5 cm. 1959. Bellman Publishing Company, Post Office Box 172, Cambridge 38, Massachusetts.

HOW TO CHOOSE A CORRESPONDENCE SCHOOL, No. 101 IN VOCATIONAL AND PROFESSIONAL MONOGRAPHS, by Homer Kempfer. Paper. Pages 35. 23×15.5 cm. 1959. Bellman Publishing Company, Post Office Box 172, Cambridge 38, Massachusetts.

THE DAWNING SPACE AGE, by H. E. Mehrens. Paper. Pages 224. 22×14 cm. 1959. Civil Air Patrol, Ellington Air Force Base, Texas. Price $2.00.

STANDARD HANDBOOK FOR TELESCOPE MAKING, by N. E. Howard. Cloth. Pages x+326. 23×15 cm. 1959. Thomas Y. Crowell Company, 423 Fourth Ave., New York 16, N. Y. Price $5.95.

READINGS IN THE LITERATURE OF SCIENCE, by William C. Dampier and Margaret Dampier. Paper. Pages 275. 20×13.5 cm. 1959. Harper Torchbooks / The Science Library, Harper & Brothers Publishers, 49 East 33rd Street, New York 16, N. Y. Price $1.50.

INTRODUCTION TO MATHEMATICAL THINKING, by Friedrich Waismann. Paper. Pages 260. 20×13.5 cm. 1959. Harper Torchbooks / The Science Library, Harper & Brothers Publishers, 49 East 33rd Street, New York 16, N. Y. Price $1.40.

SPACE, TIME AND GRAVITATION, by Sir Arthur Eddington. Paper. Pages 213. 20×13.5 cm. 1959. Harper Torchbooks / The Science Library, Harper & Brothers Publishers, 49 East 33rd Street, New York 16, N. Y. Price $1.35.

ON UNDERSTANDING PHYSICS, AN ANALYSIS OF THE PHILOSOPHY OF PHYSICS, by W. H. Watson. Paper. Pages 146. 20×13.5 cm. 1959. Harper Torchbooks / The Science Library, Harper & Brothers Publishers, 49 East 33rd Street, New York 16, N. Y. Price $1.25.

A HISTORY OF SCIENCE TECHNOLOGY & PHILOSOPHY IN THE 16TH & 17TH CENTURIES, Vol. I, II. by A. Wolf. Paper. Pages 686. 20×13.5 cm. 1959. Harper Torchbooks / The Science Library, Harper & Brothers Publishers, 49 East 33rd Street, New York 16, N. Y. Price $1.95.

CURRENT EXPENDITURES PER PUPIL IN PUBLIC SCHOOL SYSTEMS: URBAN SCHOOL SYSTEMS, 1957–58, by Gerald Kahn. Paper. Pages v+71. 20×26 cm. 1959. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price $.45.

DIMENSIONS, UNITS, AND NUMBERS IN THE TEACHING OF PHYSICAL SCIENCES, by Renée G. Ford and Ralph E. Cullman. Paper. Pages ix+49. 14×21 cm. 1959. Bureau of Publications, Teachers College, Columbia University, New York, N. Y. Price $1.00.

OPENING THE DOOR TO OPPORTUNITY, 1959–1960, General Motors Scholarships for High School Seniors, Paper. 34 Pages. 13×20 cm. 1959. General Motors Corporation, Detroit 2, Michigan.

CHEMISTRY OF NUCLEAR POWER, by J. K. Dawson, Ph.D., F. R. I. C., and G. Long, Ph.D., A.R.I.C. Cloth. 208 Pages. 13.5×21.5 cm. 1959. Philosophical Library Inc., 15 East 40th Street, New York 16, N. Y. Price $10.00.

BASIC ELECTRONICS, Vol. 6 by Van Valkenburgh, Nooger & Neville, Inc. Paper. Pages v+130. 15×23 cm. 1959. John F. Rider Publisher, Inc., 116 West 14th Street, New York 11, New York. Price $2.90.

SEEING THE EARTH FROM SPACE, Based in Part on "Man-Made Moons," by Irving Adler. Cloth. 160 Pages. 13×20.5 cm. 1959. The John Day Company, Inc., 210 Madison Avenue, New York, N. Y. Price $3.50.

UNDERSTANDING CHEMISTRY, A Brilliant Survey of Man's Conquest of Matter, by Lawrence P. Lessing. Paper. Pages vi+192. 11×18 cm. 1959. The New American Library of World Literature, Inc., 501 Madison Avenue, New York 22, New York. Price $.50.

A SHORT HISTORY OF SCIENTIFIC IDEAS TO 1900, by Charles Singer. Cloth. Pages xviii+525. 13×21.5 cm. 1959. Oxford University Press, 417 Fifth Avenue, New York 16, N. Y. Price $8.00.

THE GENIE AND THE WORD, by Walter Buehr. Cloth. 88 Pages. 15.5×21.5 cm. 1959. G. P. Putnam's Sons, 210 Madison Ave., New York, N. Y. Price $3.00.

METEORS FOUND ACTING AS MIRRORS IN THE SKY

Meteors act as "mirrors in the sky," reflecting radio waves to interfere sometimes with communications of "ham" operators and with radio sets in the home.

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Meteors are tiny bits of interplanetary debris that make "shooting stars" when they burn up in the earth's atmosphere at great speeds. The average meteor releases power at a rate of 10,000,000 watts of energy when it disintegrates in the earth's upper atmosphere. If this energy were converted into light energy, it would yield enough power to light 10,000 flood lamps of 1,000 watts each.

Book Reviews

MATHEMATICS AND THE PHYSICAL WORLD, by Morris Kline. Cloth. Pages ix+482. 15×22.5 cm. 1959. Thomas Y. Crowell Company, 432 Fourth Avenue, New York, 16, N. Y. Price $6.00.

Throughout this book one seems to sense a criticism of those mathematicians who are not interested in science and applications of mathematics, but prefer to study mathematics for its own sake. It is indeed hard to classify this book—the approach is to a marked degree historical, yet there is much space devoted to developments or proofs. As the author states, we see in some places mathematics being born; at others we see how mathematics provides answers to physical problems, and why mathematics has become the essence of scientific theories.

The author claims that the presentation is confined to elementary mathematics; certainly this is true in the first portion of the book. In fact, the mathematical work seems excessively detailed to one with even an elementary background. When in later chapters one encounters the calculus of variations, differential equations, and non-Euclidean geometries, it is indeed questionable whether some readers may not have been lost by the wayside.

This reviewer feels that many high school students (and teachers!) will be intrigued by some of the material in this book. As examples, one might cite the proofs, both by very elementary algebra and by elementary geometry, that the area of a square is greater than that of any other rectangle with equal perimeter. Other maximum problems are treated in an elementary manner, for example, it is proved that a circle has greater area than a square of the same perimeter. Another concept, developed by elementary methods, is the reflective property of the parabola. Many boys will no doubt be interested in the analysis of the problem of firing a shell from a gun so as to hit a bomb which has been dropped from an airplane. These are merely illustrative of a great number of topics.

Those who are trying to teach students to understand the *why* of mathematics will no doubt object to such statements as: "The method is not obvious and yet a very simple idea does the trick." (p. 69); "Without pretending to understand in the slightest the rationale of this procedure, let us carry it out." (p. 390); "John Bernoulli solved this problem by a trick . . . " (p. 426).

The reviewer was very pleased with the first portion of the book. although in spots the mathematical derivations seemed unduly prolonged and somewhat tedious. This was considered excusable if the book is addressed to the individual with small mathematical background. As the calculus is introduced (not always with complete rigor) and the development went farther into what might be termed advanced mathematics, the feeling developed that here is an attempt to present a major portion of mathematics in a single volume. Even if the reader persists to the end, will he have a false idea of his competence? It seems a little odd to be seeking a maximum of the product sin A cos A before introducing the meaning of trigonometric functions of angles greater than a right angle. Indicating a derivative by placing a dot over the variable may have historic importance, but it might be worth mentioning that this is not now commonly used. Guass seems to be given more credit for the invention of non-Euclidean geometry than some authorities would grant.

The book is interesting reading, teachers and students alike could profit from many portions. Perhaps here is a case where the young reader might have some guidance in his reading, rather than be turned loose with the book without any comments or suggestions. If the non-mathematician will take the trouble to read the material carefully, not merely skim over it, he may find answers to many questions in which he has been interested.

CECIL B. READ
University of Wichita

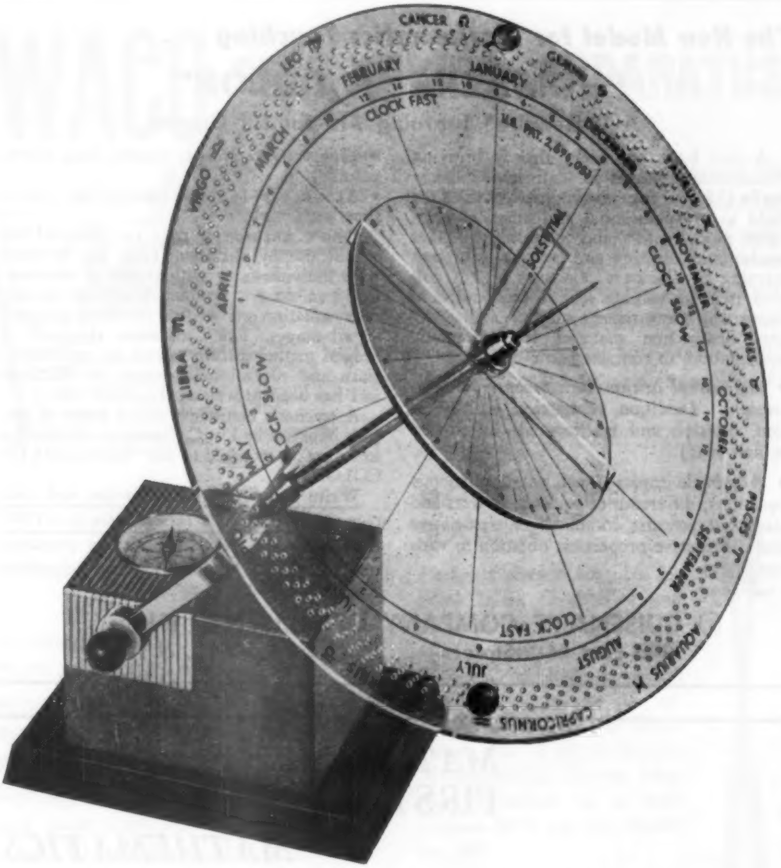
ON MATHEMATICS AND MATHEMATICIANS, by Robert Edouard Moritz. Paper. Pages vii+410. 13.5×20.5 cm. 1914, 1942. Dover Publications, Inc., 920 Broadway, New York 10, N. Y. Price $1.95.

This is a republication in unaltered form of a classic which was first issued some forty-five years ago under the title "Memorabilia Mathematica or the Philomath's Quotation Book." Anyone who has not had access to this reference and hence has spent long hours searching for the exact wording or the original source of some half-remembered quotation will indeed appreciate the fact that this work is now available in an inexpensive form.

The book consists of quotations about mathematics and by and about mathematicians. The various chapters group the material roughly into twenty-one categories, the index covers several hundred topics. The reviewer was interested in seeing how successful one might be in locating certain quotations—as examples Newton's statement about playing on the beach and finding a prettier shell or smoother pebble, and the familiar statement that there is no royal road to geometry. There was no difficulty at all in finding these and similar passages.

Certainly this book is a splendid source of material for the teacher, the writer, or the lecturer. Many of the quotations would form excellent material for classroom bulletin board display.

CECIL B. READ
University of Wichita



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